SOCI 620: Quantitative methods 2

Feb 14

Expanding on Poisson regressions

1. Administrative
2. Interpreting coefficients from Poisson regressions
3. Over-dispersed Poisson regressions
4. Zero-inflated Poisson regressions
5. Over-dispersion and zero-inflation in R

Interpreting coefficients

Model from last time

$$H_i \sim \text{Pois}(\lambda_i)$$

$$\log(\lambda_i) = \alpha + \beta_M M_i + \beta_G G_i$$

$$\alpha \sim \text{Norm}(3, 1)$$

$$\beta_M \sim \text{Norm}(0, 0.5)$$

$$\beta_G \sim \text{Norm}(0, 0.3)$$

	Mean	exp(Mean)
α	0.27	1.32
β_M	1.11	3.05
β_G	-0.14	0.87

α (baseline)

A student who is not a boy (M_i=0) and is in grade 10 (G_i=0) is predicted to play about 1.32 hours of games per week.

β_M (gender)

Boys (M_i=1) are expected to spend about 3.05 times more time than non-boys (M_i=0) playing games.

β_G (grade)

A one-year increase in grade is associated with playing 0.87 times as much. This is a decrease of 13% each year.

Over-dispersion

$$H_i \sim \text{Pois}(\lambda_i)$$

$$\log(\lambda_i) = \alpha + \beta M_i$$

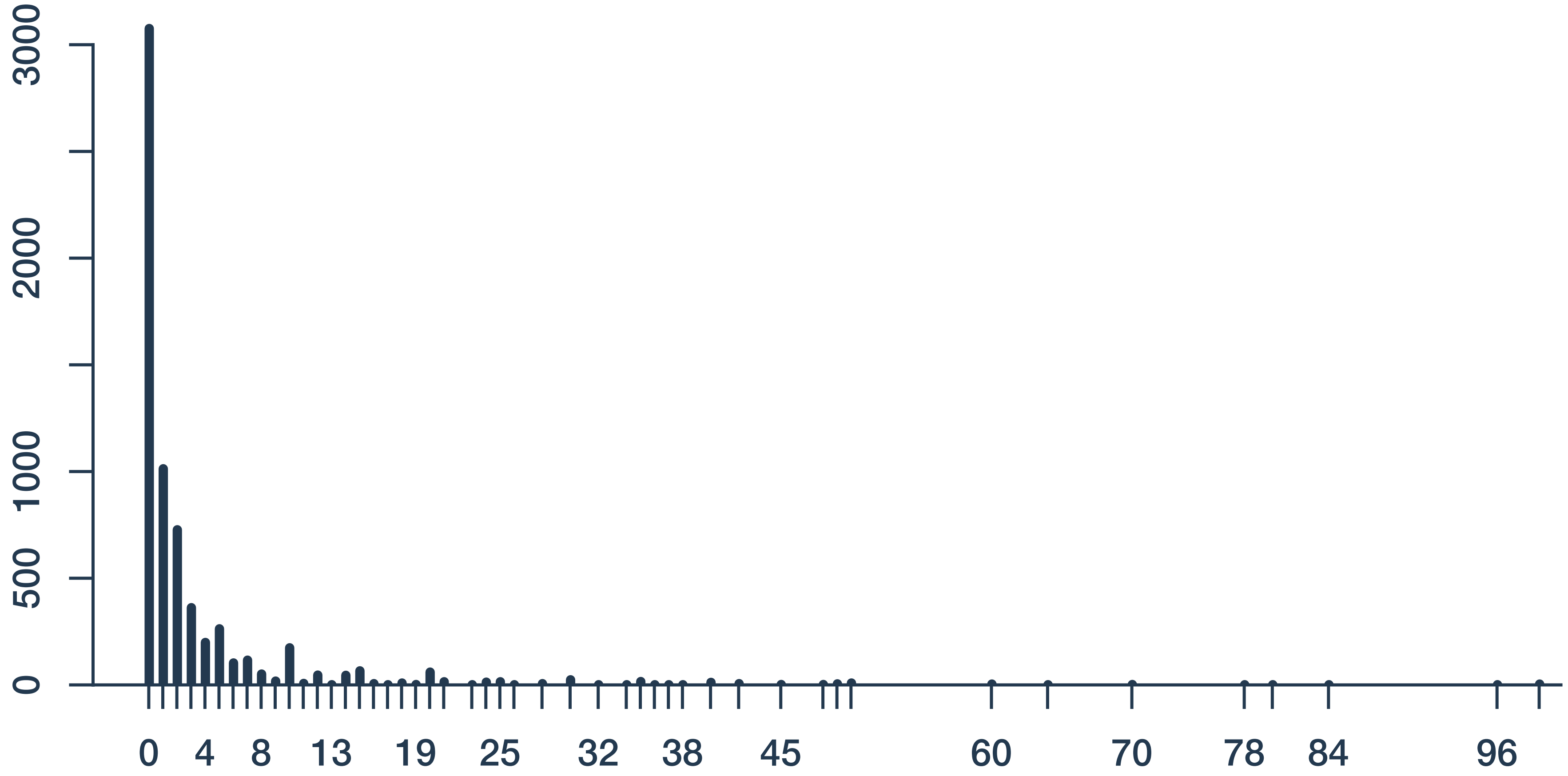
$$\alpha \sim \text{Norm}(3, 0.5)$$

$$\beta \sim \text{Norm}(0, 0.3)$$

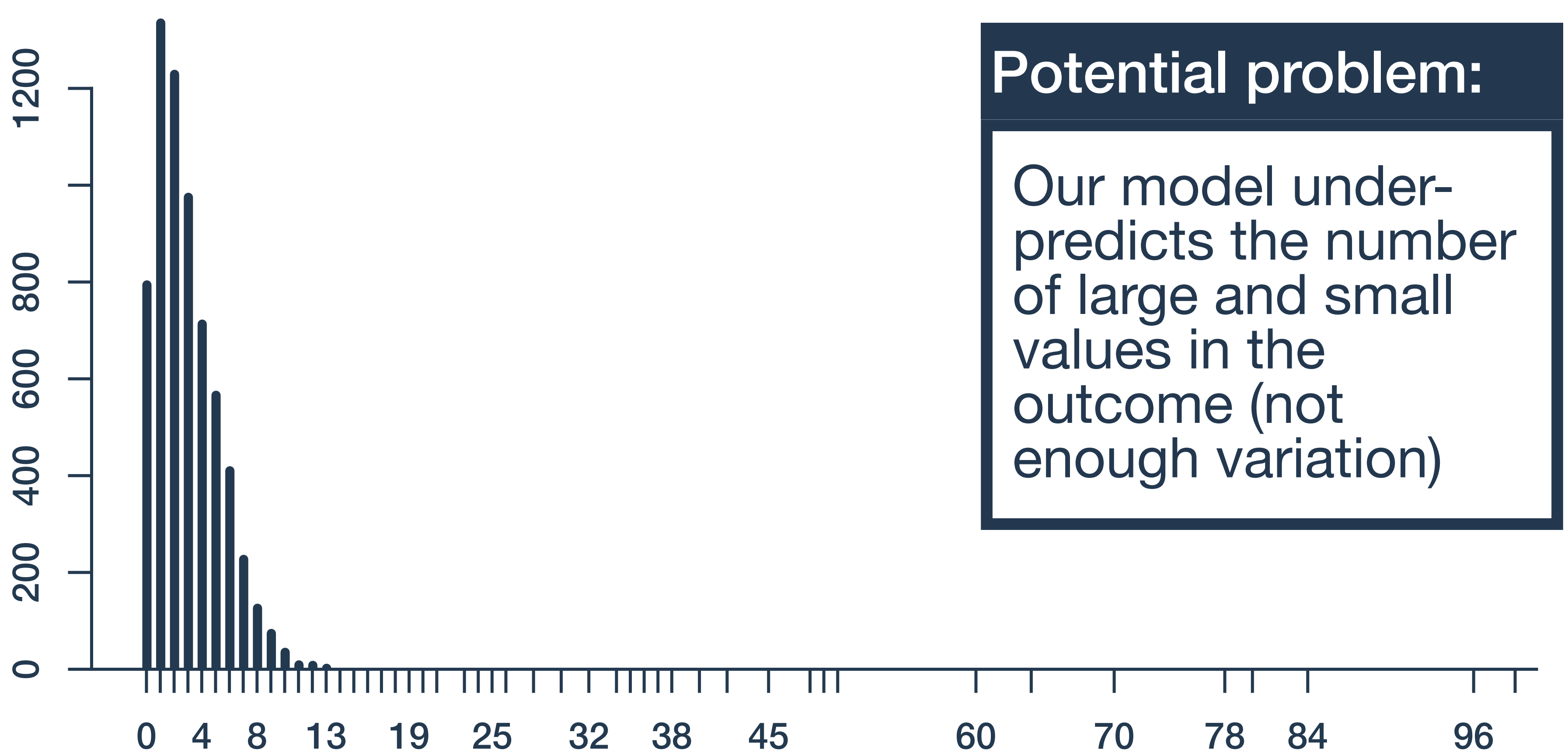
	Mean	exp(Mean)
α	0.27	1.32
β	1.11	3.05

Over-dispersion

Actual distribution



Posterior predicted distribution (Poisson regression)



Potential problem:

Our model under-predicts the number of large and small values in the outcome (not enough variation)

Over-dispersion

Poisson regression

$$
\begin{aligned}
H_i &\sim \text{Pois}(\lambda_i) \\
\log(\lambda_i) &= a + \beta M_i \\
a &\sim \text{Norm}(3, 0.5) \\
\beta &\sim \text{Norm}(0, 0.3)
\end{aligned}
$$

Gamma-Poisson regression

$$
\begin{aligned}
H_i &\sim \text{Pois}(\lambda_i) \\
\lambda_i &\sim \text{Gamma}(\mu_i, \theta) \\
\log(\mu_i) &= a + \beta M_i \\
a &\sim \text{Norm}(3, 0.5) \\
\beta &\sim \text{Norm}(0, 0.3) \\
\theta &\sim \text{Unif}(0, 10)
\end{aligned}
$$

Extra "dispersion" from gamma (on $\lambda_i \sim \text{Gamma}(\mu_i, \theta)$)

Two students who look identical based on covariates can have different Poisson rates λ_i.

One more prior (on $\theta \sim \text{Unif}(0, 10)$)

A.K.A.

Negative-binomial regression

Over-dispersed Poisson regression

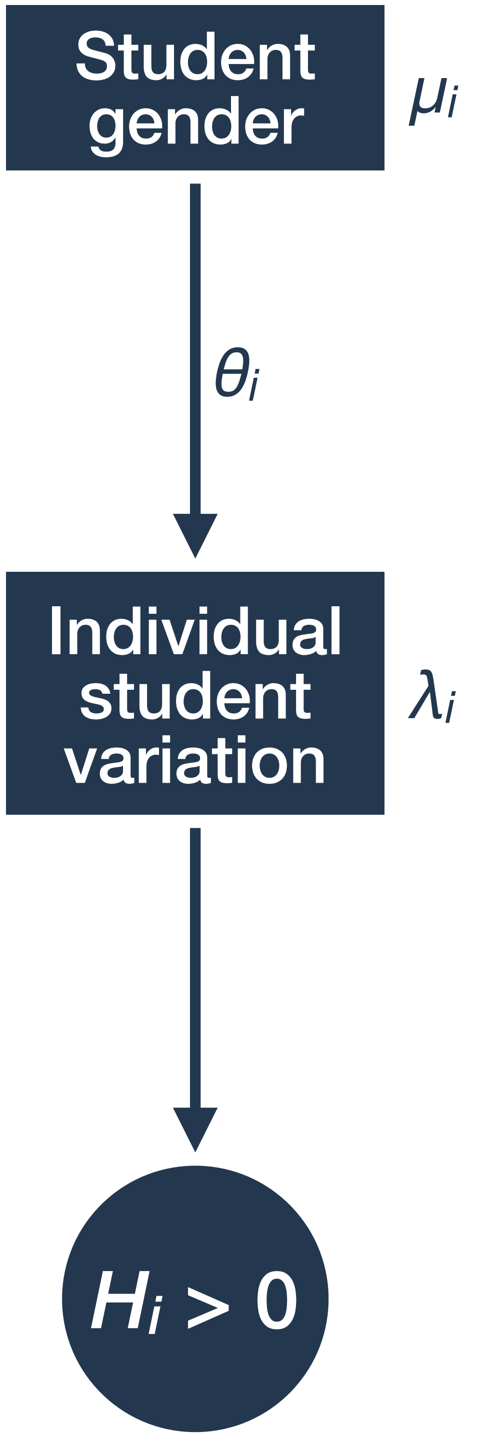
Over-dispersion

Gamma-Poisson regression

$$H_i \sim \text{Pois}(\lambda_i)$$
$$\lambda_i \sim \text{Gamma}(\mu_i, \theta)$$
$$\log(\mu_i) = \alpha + \beta M_i$$

$$\alpha \sim \text{Norm}(3, 0.5)$$
$$\beta \sim \text{Norm}(0, 0.3)$$
$$\theta \sim \text{Unif}(0, 10)$$

Data story:



Over-dispersion

$$H_i \sim \text{GammaPois}(\lambda_i, \theta)$$
$$\log(\lambda_i) = \alpha + \beta M_i$$

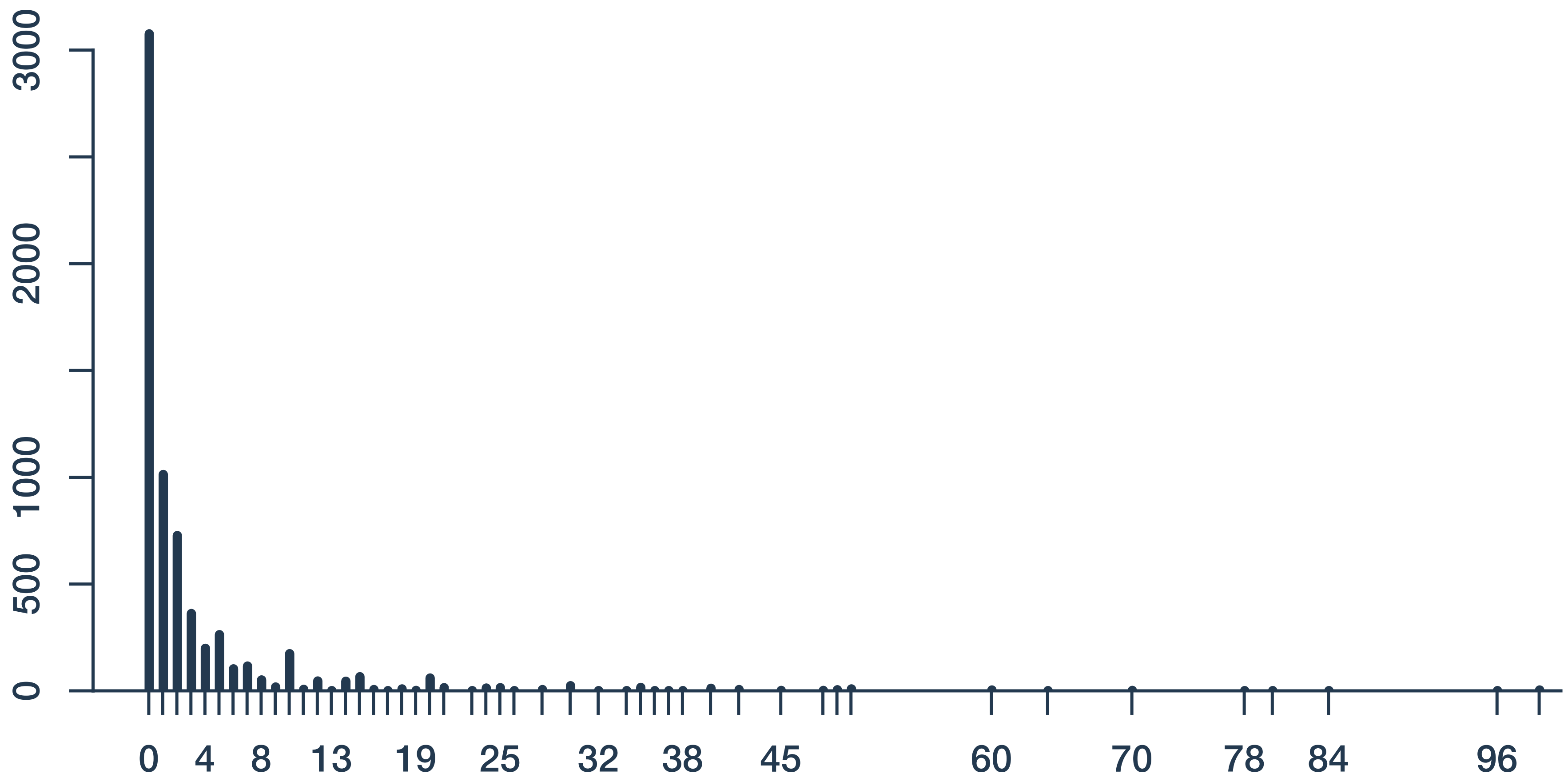
$$\alpha \sim \text{Norm}(3, 0.5)$$
$$\beta \sim \text{Norm}(0, 0.3)$$
$$\theta \sim \text{Unif}(0, 10)$$

	Mean	*90% credible interval*		exp(Mean)
α	0.55	0.50	0.61	1.74
β	0.86	0.80	0.91	2.35
θ	8.56	8.09	9.03	—

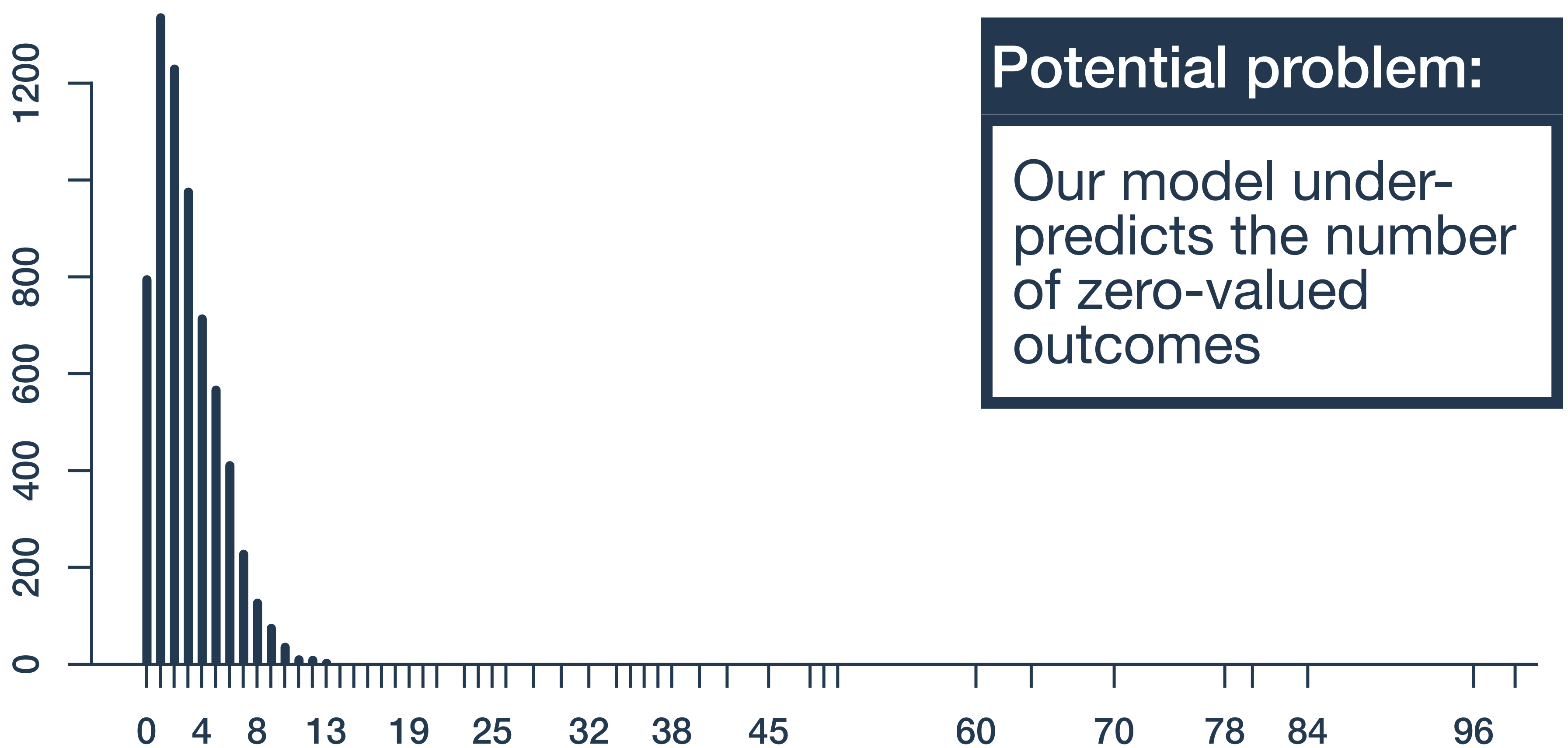
θ measures extra dispersion →

Zero-inflation

Actual distribution



Posterior predicted distribution (Poisson regression)



Potential problem:

Our model under-predicts the number of zero-valued outcomes

Zero-inflation

Outcome variable is result of one of two processes

Either the student does not own a game console ($c_i = 1$) **or** the student does own a console and plays at some rate λ_i ($c_i = 0$).

$$H_i \begin{cases} = 0 & \text{if } c_i = 1 \\ \sim \text{Pois}(\lambda_i) & \text{if } c_i = 0 \end{cases}$$

Zero-inflation

Their chance of owning a console is modeled with p_i

$$H_i \begin{cases} = 0 & \text{if } c_i = 1 \\ \sim \text{Pois}(\lambda_i) & \text{if } c_i = 0 \end{cases}$$

$$c_i \sim \text{Bern}(p_i)$$

Zero-inflation

p_i is modeled as a linear function of family income

$$H_i \begin{cases} = 0 & \text{if } c_i = 1 \\ \sim \text{Pois}(\lambda_i) & \text{if } c_i = 0 \end{cases}$$

$$c_i \sim \text{Bern}(p_i)$$

$$\text{logit}(p_i) = \alpha_p + \beta_p W_i$$

Zero-inflation

λ_i is modeled as a linear function of gender

$$H_i \begin{cases} = 0 & \text{if } c_i = 1 \\ \sim \text{Pois}(\lambda_i) & \text{if } c_i = 0 \end{cases}$$

$$c_i \sim \text{Bern}(p_i)$$

$$\text{logit}(p_i) = a_p + \beta_p W_i$$

$$\log(\lambda_i) = a_\lambda + \beta_\lambda M_i$$

Zero-inflation

All four parameters need priors →

$$H_i \begin{cases} = 0 & \text{if } c_i = 1 \\ \sim \text{Pois}(\lambda_i) & \text{if } c_i = 0 \end{cases}$$

$$c_i \sim \text{Bern}(p_i)$$

$$\text{logit}(p_i) = \alpha_p + \beta_p W_i$$

$$\log(\lambda_i) = \alpha_\lambda + \beta_\lambda M_i$$

$$\alpha_p \sim \text{Norm}(0, 1)$$

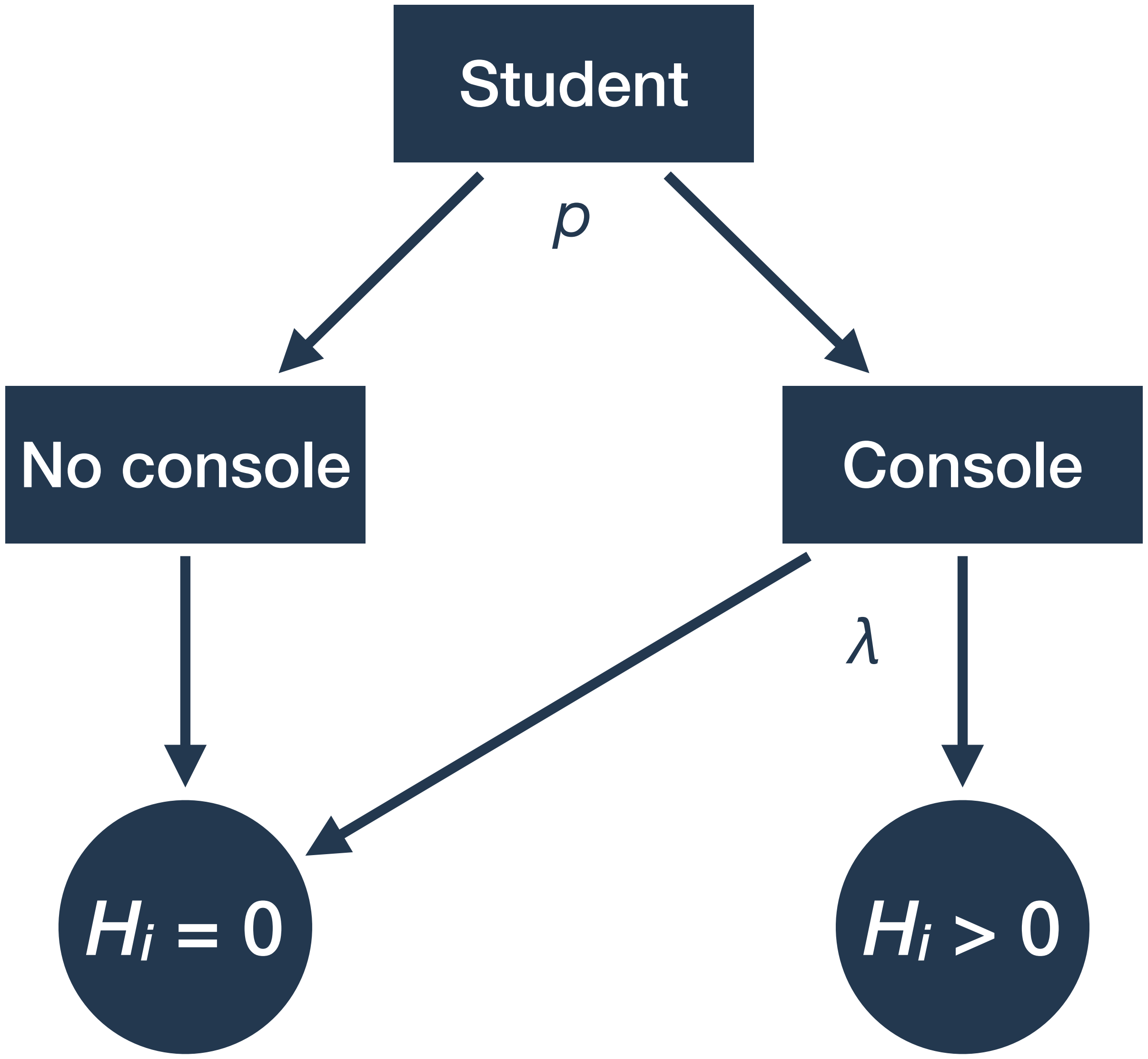
$$\beta_p \sim \text{Norm}(0, 2)$$

$$\alpha_\lambda \sim \text{Norm}(3, 0.5)$$

$$\beta_\lambda \sim \text{Norm}(0, 0.3)$$

Zero-inflation

Data story:



$$H_i \begin{cases} = 0 & \text{if } c_i = 1 \\ \sim \text{Pois}(\lambda_i) & \text{if } c_i = 0 \end{cases}$$

$$c_i \sim \text{Bern}(p_i)$$

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