AGENDA

Counts & rates

1. Counts as random outcomes
2. The log link function
3. ***Hands on:*** *Poisson models in R*

Counts as random outcomes

The Poisson distribution

COUNTS AS OUTCOMES

Kinds of counts:

Average and deviation

An event that is technically a count, but the *scale* of the process means we can treat it as continuous

E.g. immigration rate, unemployment rate, etc.

Normal distribution

Trials and probability of success

Outcome could have happened at most N times, our data measures how many times it did happen

E.g. "how many days per week...", etc.

Binomial / Bernoulli distribution

Rate of occurence

An event that has no (theoretical) upper limit, but tends to happen at a relatively low rate

E.g. individual fertility, number of friends, grocery stores in a neighborhood, etc.

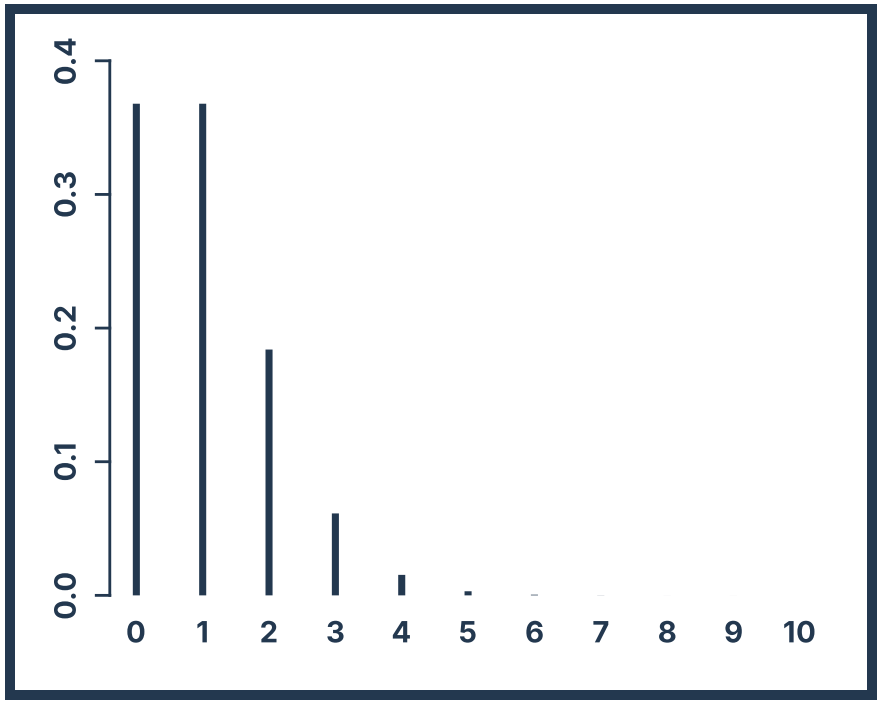
Poisson distribution

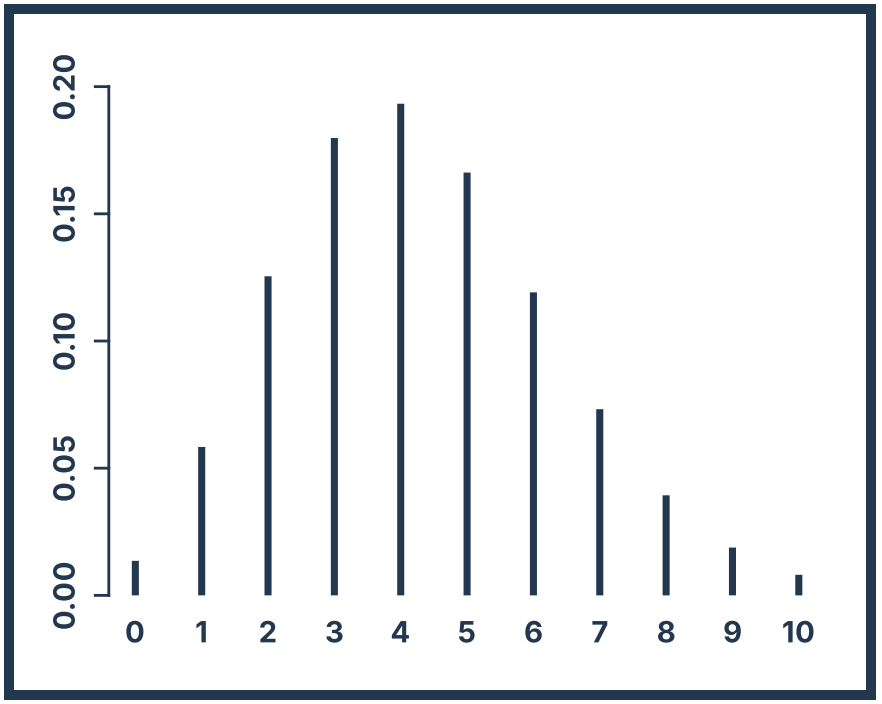
*(**See also**: geometric distribution for 'time until occurrence' types of models)*

POISSON DISTRIBUTION

The ***Poisson distribution*** gives the probability that an event will happen *k* times in a particular unit of time or space if it has an average rate of occurrence of λ in that unit of time or space.

$$\text{Prob}(k|\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

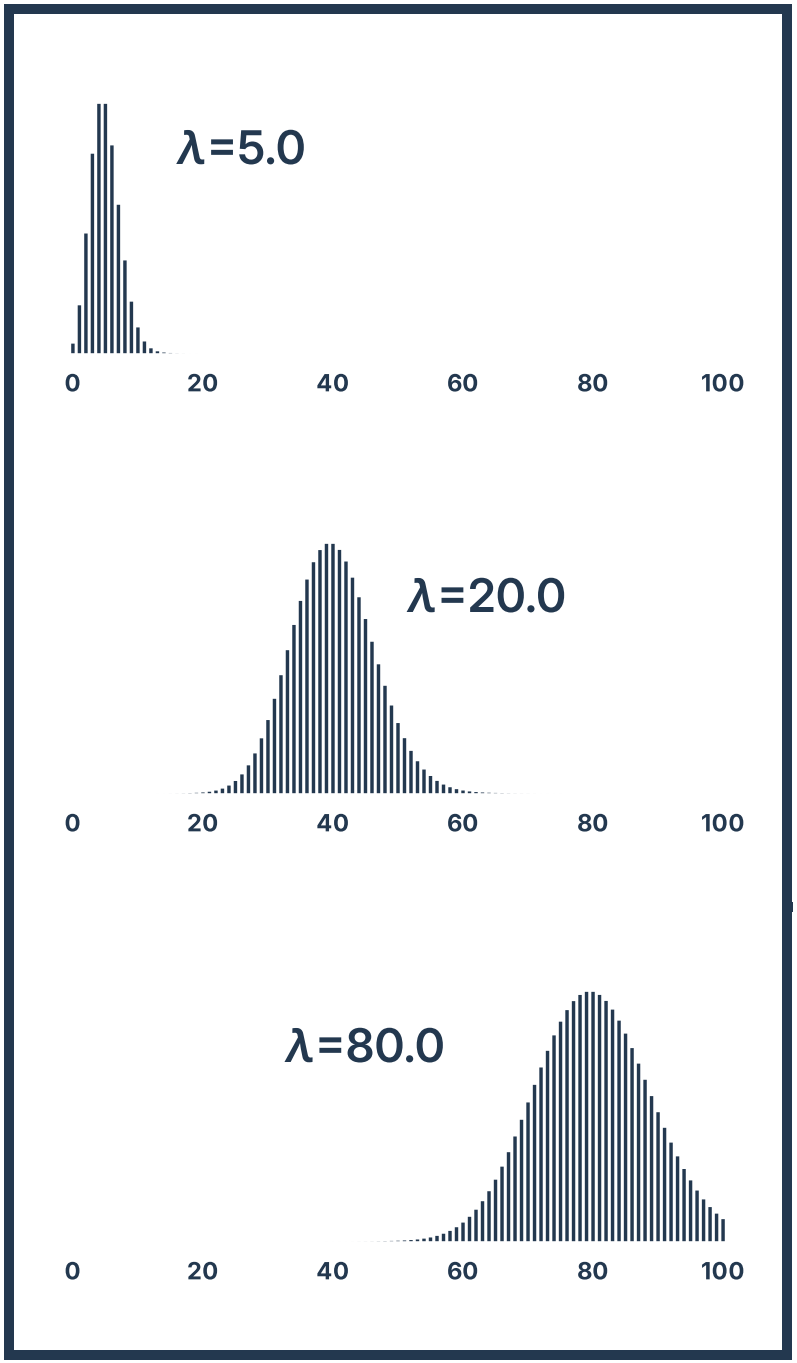




POISSON DISTRIBUTION

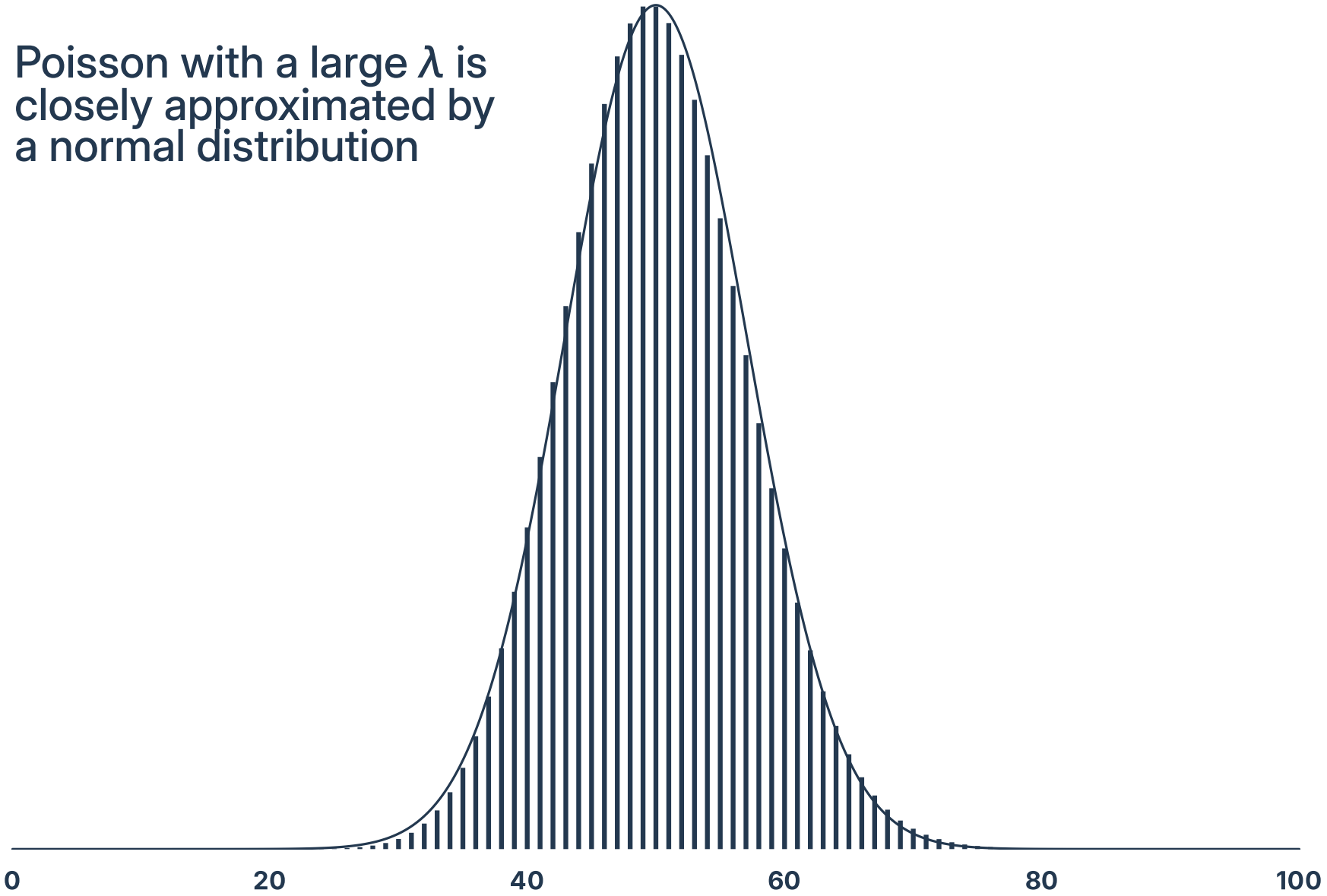
$$k \sim \mathrm{Pois}(\lambda)$$

Single parameter:
λ is both the *mean* and *variance* (s. d. squared) of the Poisson distribution



POISSON DISTRIBUTION

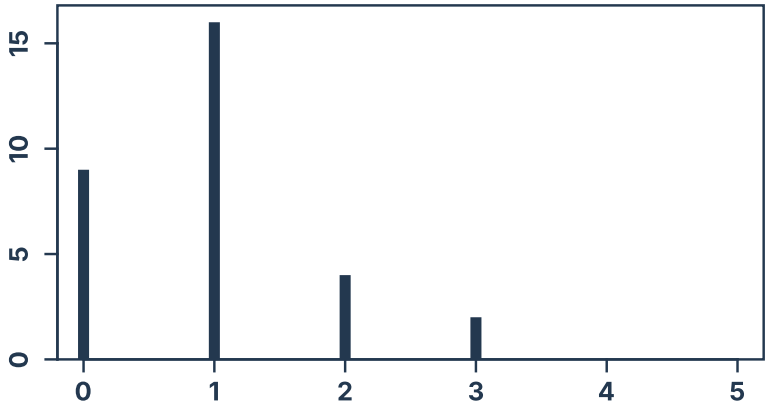
Poisson with a large λ is closely approximated by a normal distribution



POISSON-DISTRIBUTED OUTCOME

How many cups of coffee does Special Agent Dale Cooper drink in an episode of *Twin Peaks*?

(Seasons 1 and 2)

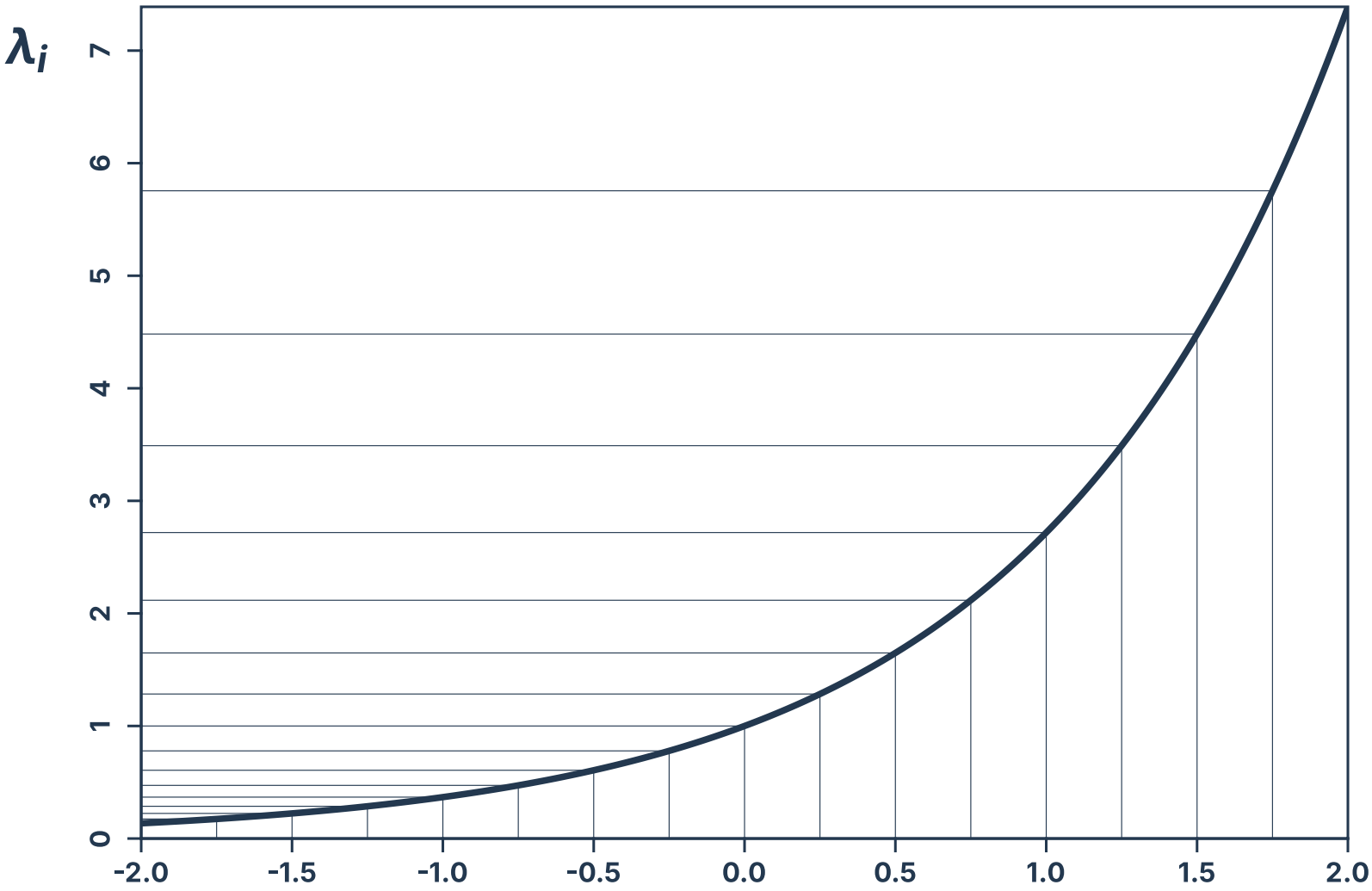


$$C_i \sim \text{Pois}(\lambda_i)$$

$$f(\lambda_i) = \alpha + \beta X_i + \dots$$

The log link

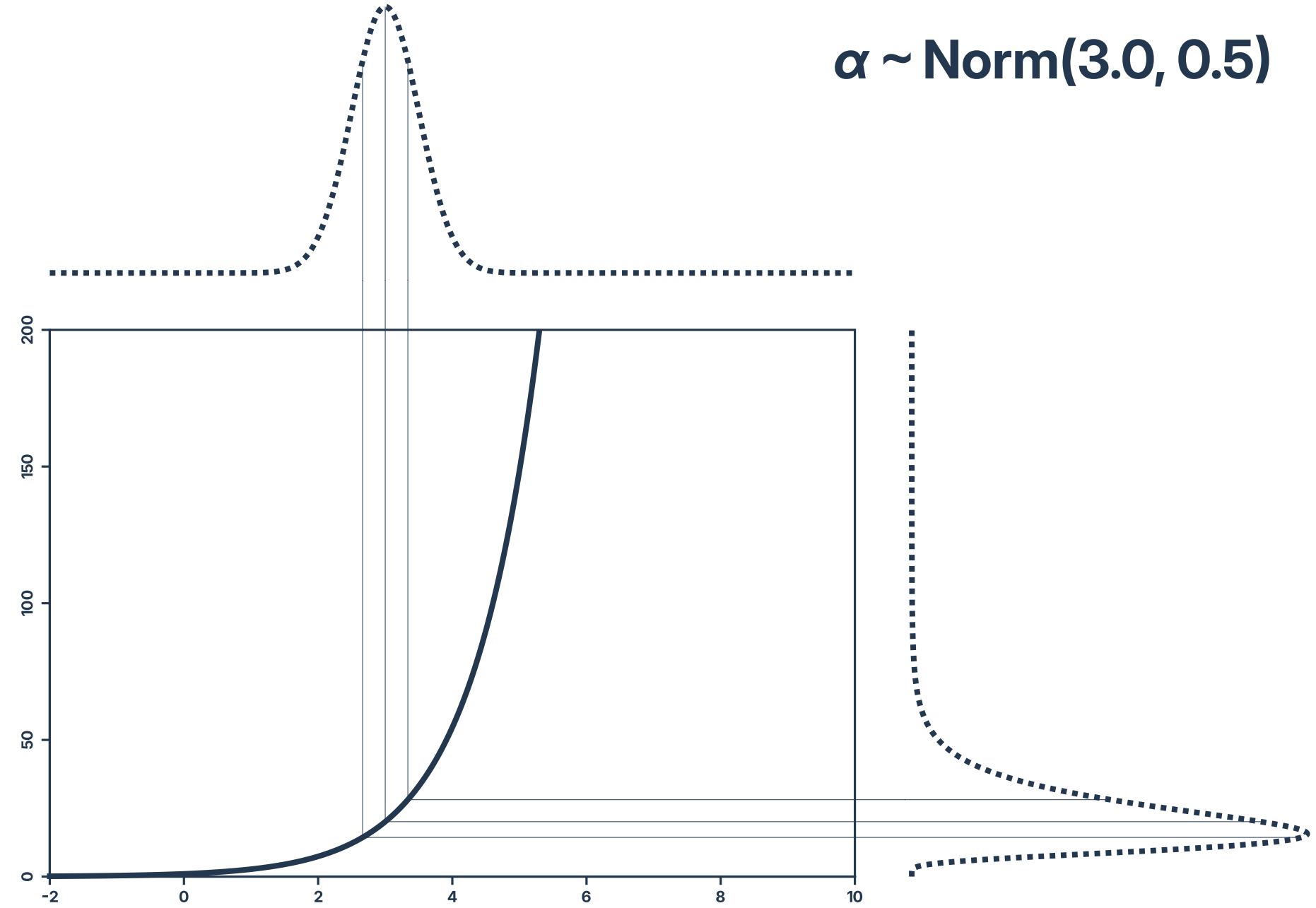
LOG LINK FUNCTION



$$\log(\lambda_i) = \alpha + \beta X_i + \ldots$$

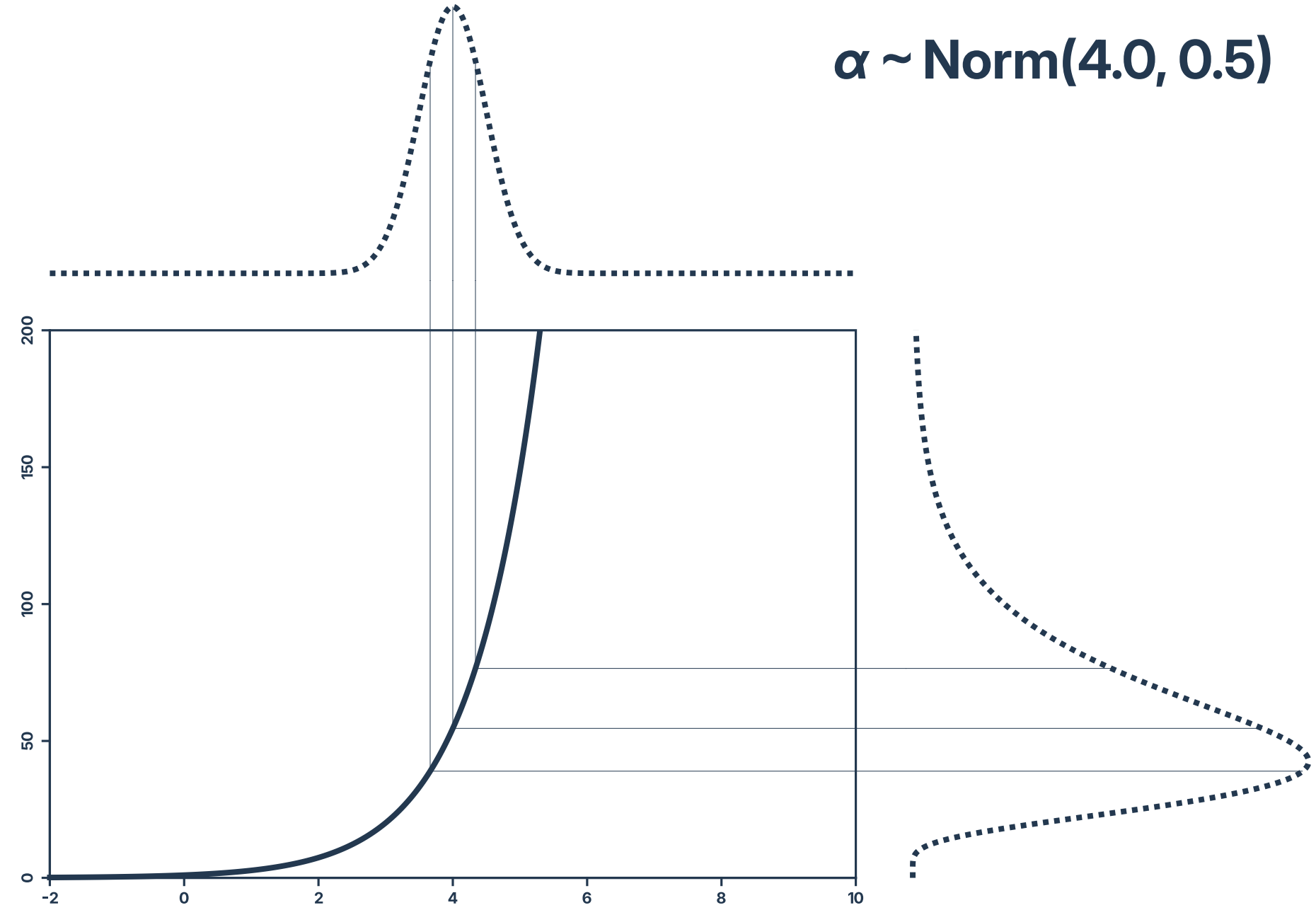
LOG LINK PRIORS

$\alpha \sim \text{Norm}(3.0, 0.5)$

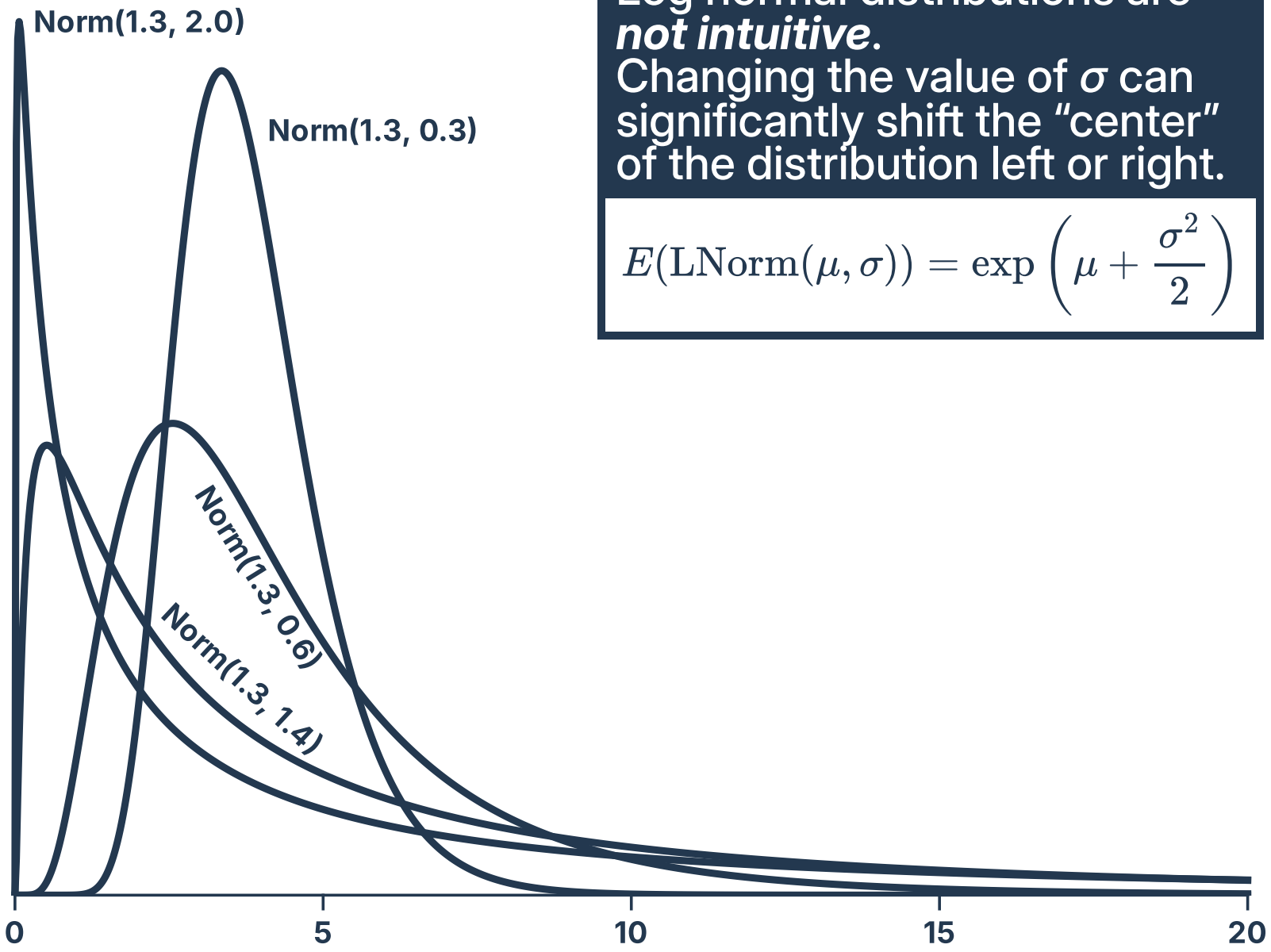


LOG LINK PRIORS

$\alpha \sim \text{Norm}(4.0, 0.5)$



LOG LINK PRIORS



Log normal distributions are ***not intuitive***.
Changing the value of σ can significantly shift the "center" of the distribution left or right.

$$E(\text{LNorm}(\mu, \sigma)) = \exp\left(\mu + \frac{\sigma^2}{2}\right)$$

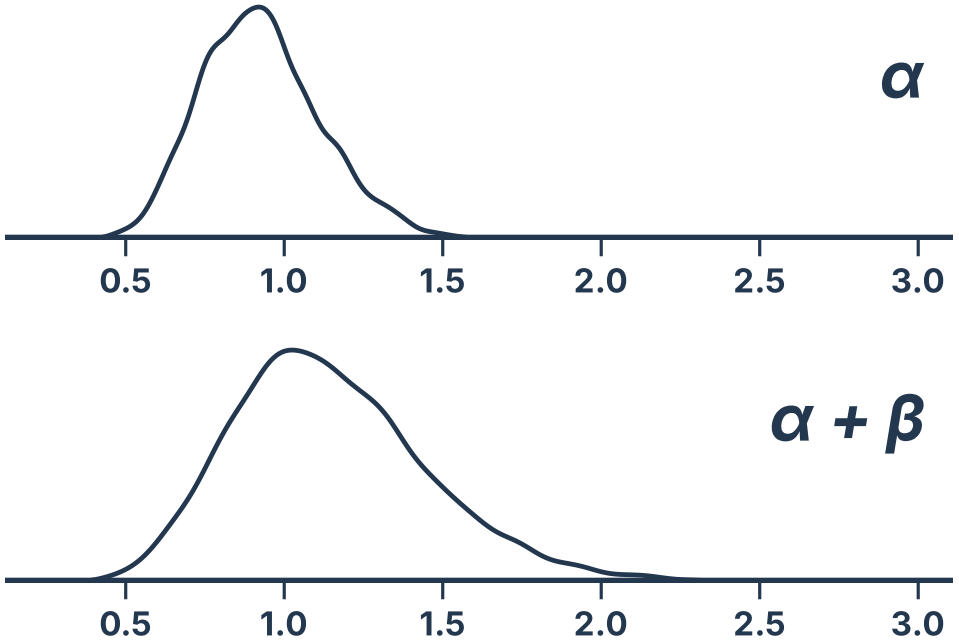
BACK TO THE MODEL

$$C_i \sim \text{Pois}(\lambda_i)$$
$$\log(\lambda_i) = \alpha + \beta S_i$$

$$\alpha \sim \text{Norm}(1.3, 1.4)$$
$$\beta \sim \text{Norm}(0, 0.5)$$

S_i – indicator for season 1

	Mean	95% C.I.
α	-0.097	(-0.513, 0.291)
β	0.200	(-0.414, 0.809)
$\exp(\alpha)$	0.908	(0.599, 1.337)
$\exp(\beta)$	1.221	(0.661, 2.247)

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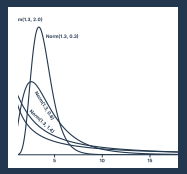

Figures by Peter McMahan (source code)

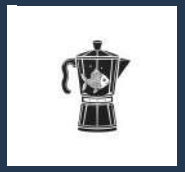

Image by 4TinyCats

Still from Twin Peaks (1990)

Promotional image for Twin Peaks (1990)

Still from Twin Peaks (2017)

Nintendo ad via Reddit