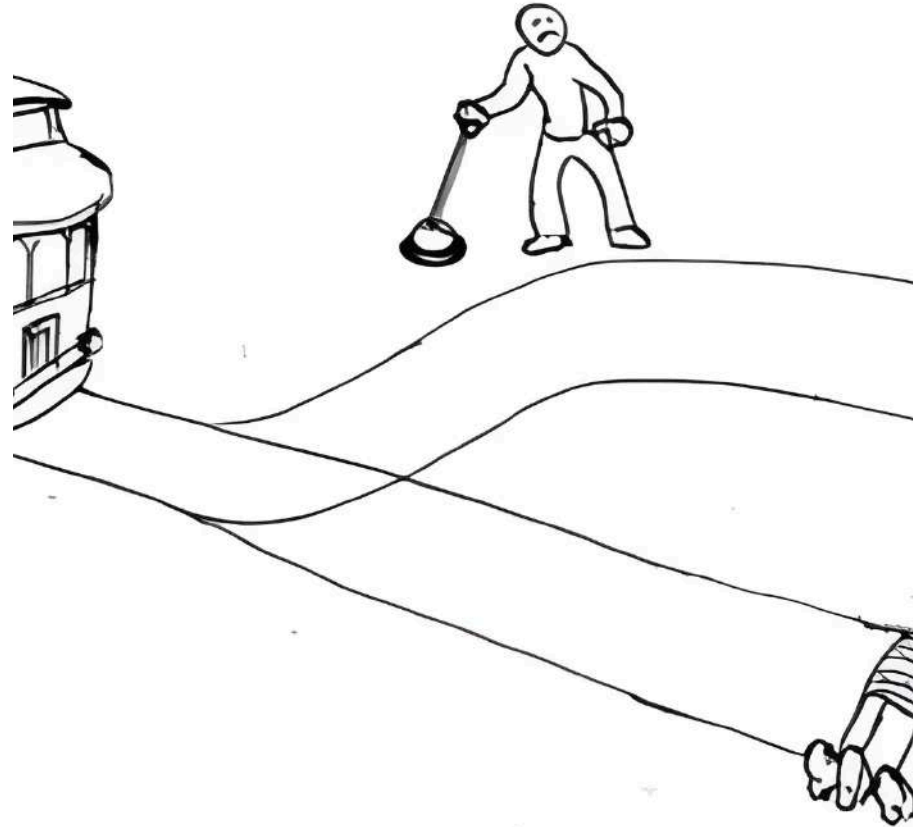
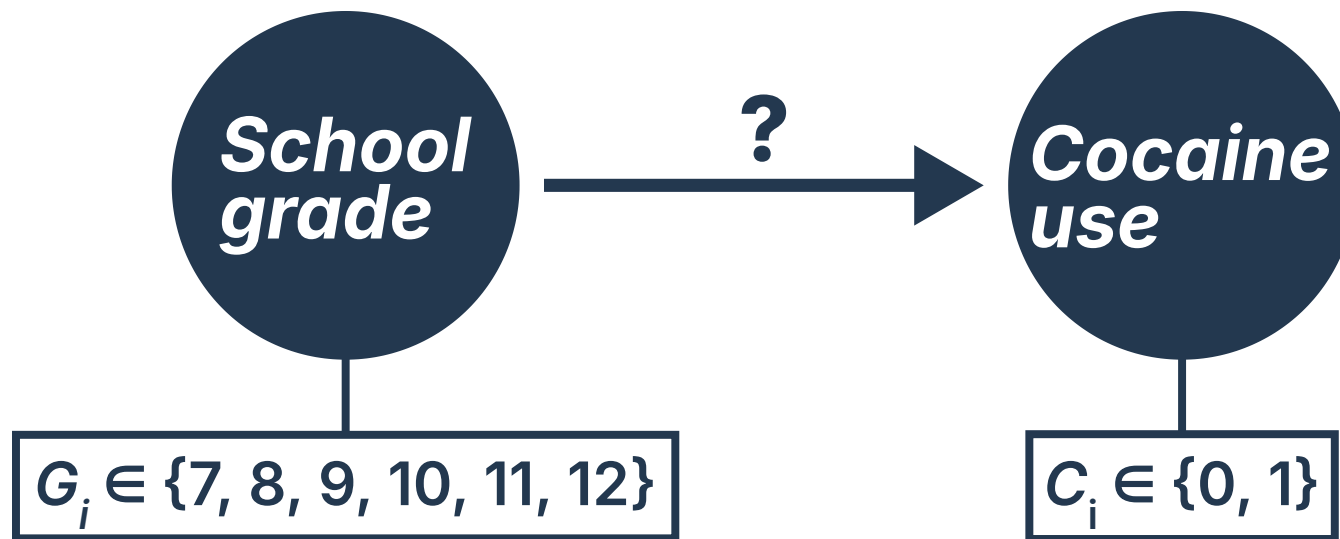


- | | |
|---------------|---|
| Agenda | 1. Logistic with predictors |
| Parsimony | 2. Interpreting coefficients |
| & overfitting | 3. <i>Hands on:</i> |
| | <i>Prior predictive simulation in R</i> |

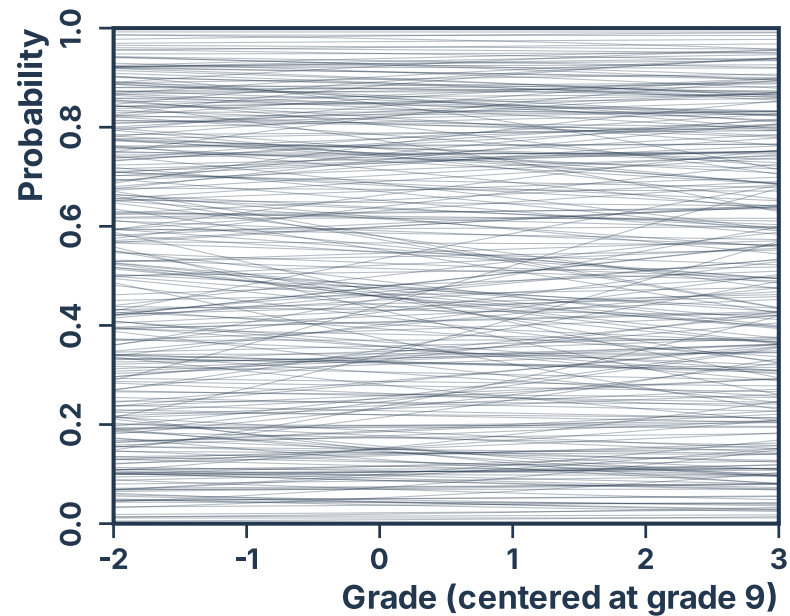
Logistic with predictors





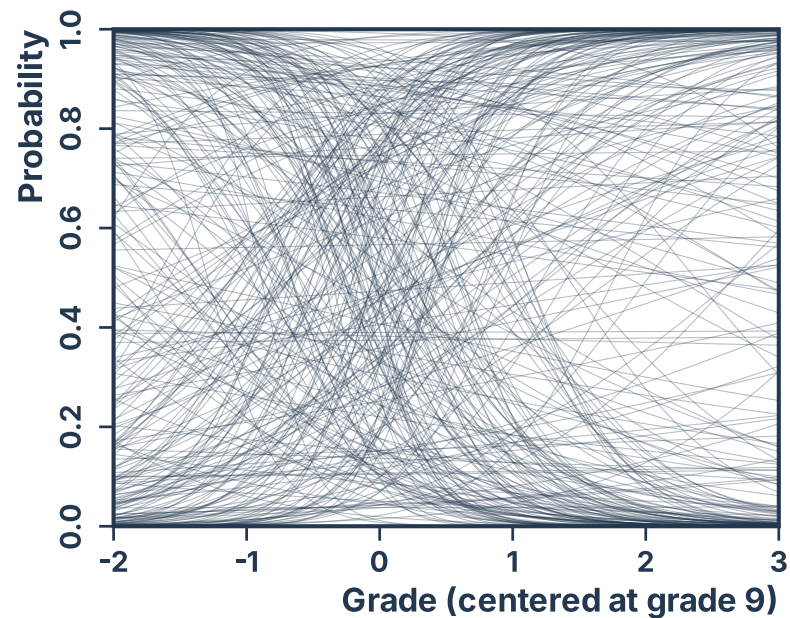
$$C_i \sim \text{Bernoulli}(p_i)$$
$$\text{logit}(p_i) = \alpha + \beta G_i$$

$$\alpha \sim \text{Norm}(0, 1.7)$$
$$\beta \sim \text{Norm}(0, ???)$$



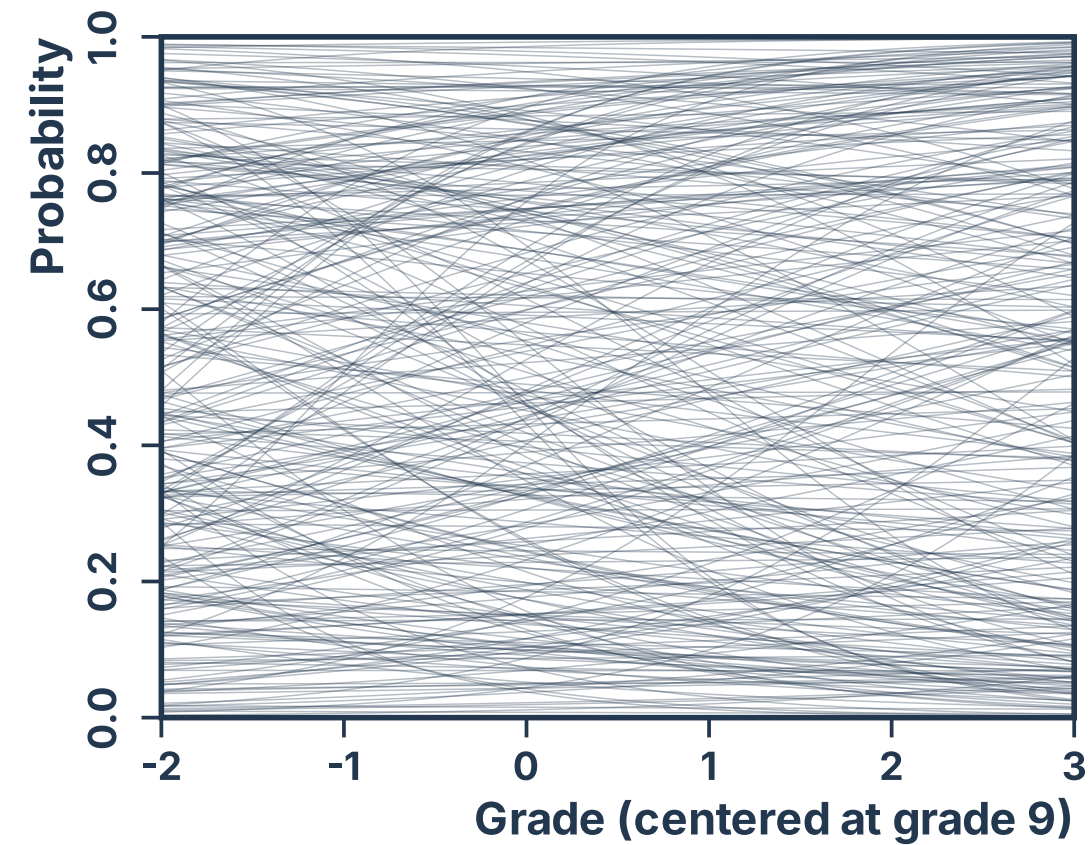
$$\alpha \sim \text{Norm}(0, 1.7)$$

$$\beta \sim \text{Norm}(0, \mathbf{0.1})$$



$$\alpha \sim \text{Norm}(0, 1.7)$$

$$\beta \sim \text{Norm}(0, \mathbf{2.0})$$

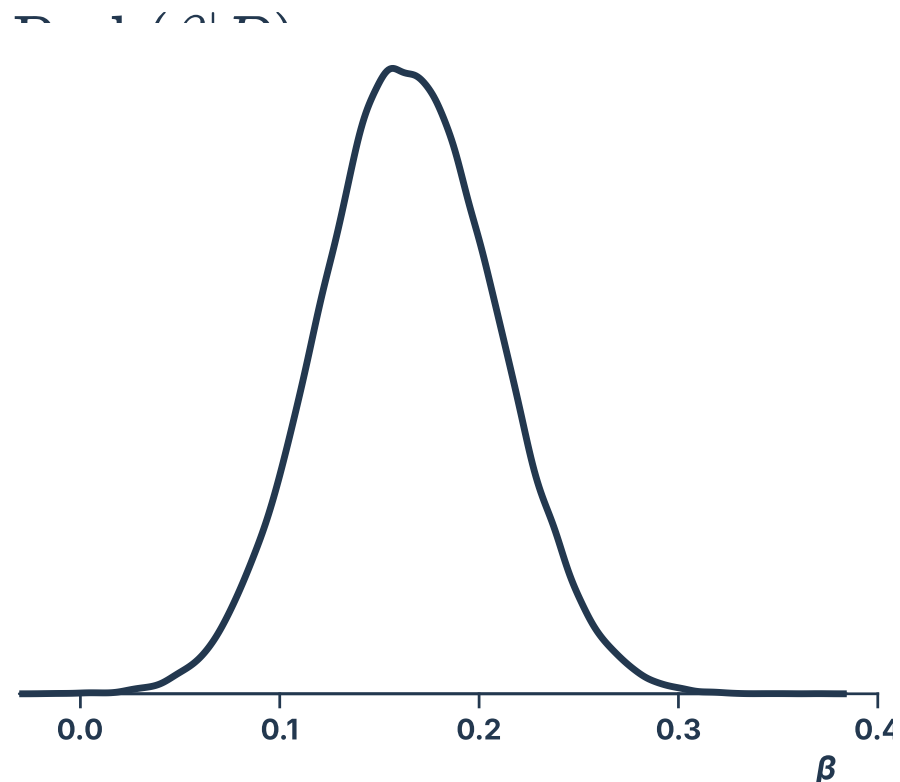
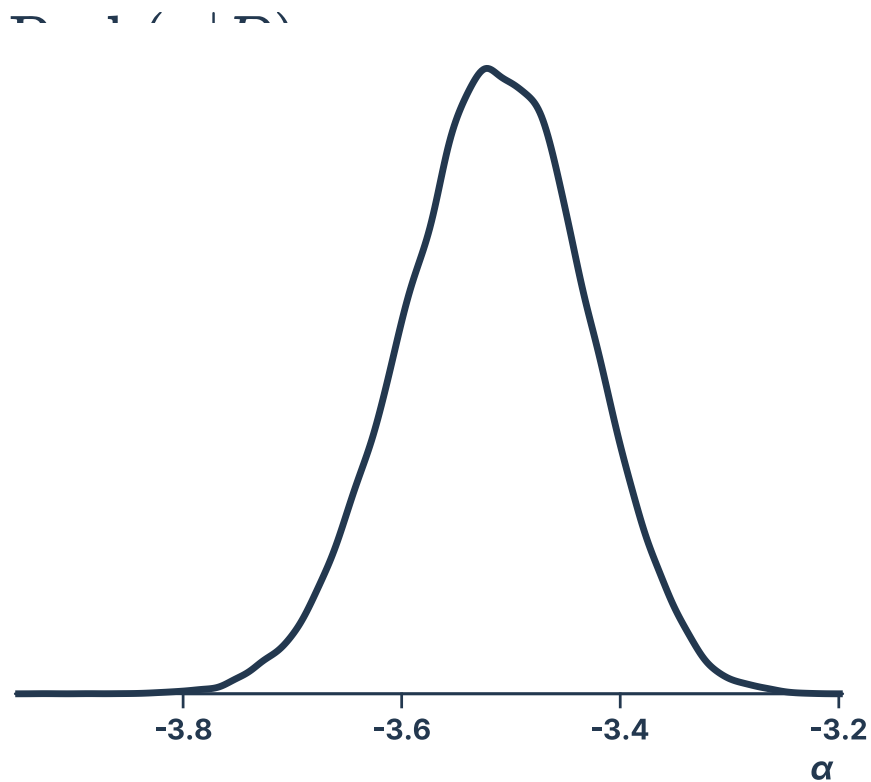


$$\alpha \sim \text{Norm}(0, 1.7)$$

$$\beta \sim \text{Norm}(0, \mathbf{0.4})$$

$$C_i \sim \text{Bernoulli}(p_i)$$
$$\text{logit}(p_i) = \alpha + \beta G_i$$

	Mean	95% C.I.
α	-3.52	(-3.68, -3.46)
β	0.16	(0.08, 0.25)



Interpreting logistic regression coefficients



$$C_i \sim \text{Bernoulli}(p_i)$$

$$\text{logit}(p_i) = \alpha + \beta G_i$$

	Mean	95% C.I.
α	-3.52	(-3.68, -3.46)
β	0.16	(0.08, 0.25)

Intercept (α)

- ∴ The expected probability of having tried cocaine for a student in grade 9 ($G_i = 0$) is:

$$\text{logit}^{-1}(-3.52) = 0.029 = 2.9\%$$

- ∴ The probability that a student in grade 9 has tried cocaine is **most likely between 2.5% and 3.4% (95% CI)**.

Coefficient (β)

- ∴ Coefficients are not so simple!
- ∴ Some options:

Odds ratio

Selected cases

Average marginal effect (AME)

Posterior visualization

Odds ratio | The ***odds ratio*** is the proportional change in the *odds* of the outcome associated with a one-unit change in the predictor.

$$\frac{\left(\frac{p^{G=1}}{1-p^{G=1}} \right)}{\left(\frac{p^{G=0}}{1-p^{G=0}} \right)} = \exp(\beta)$$

$$\begin{aligned} \text{logit}(p) &= \alpha + \beta G \\ \log\left(\frac{p}{1-p}\right) &= \alpha + \beta G \\ \frac{p}{1-p} &= \exp(\alpha + \beta G) \\ \frac{p}{1-p} &= \exp(\alpha) \times \exp(\beta G) \end{aligned}$$

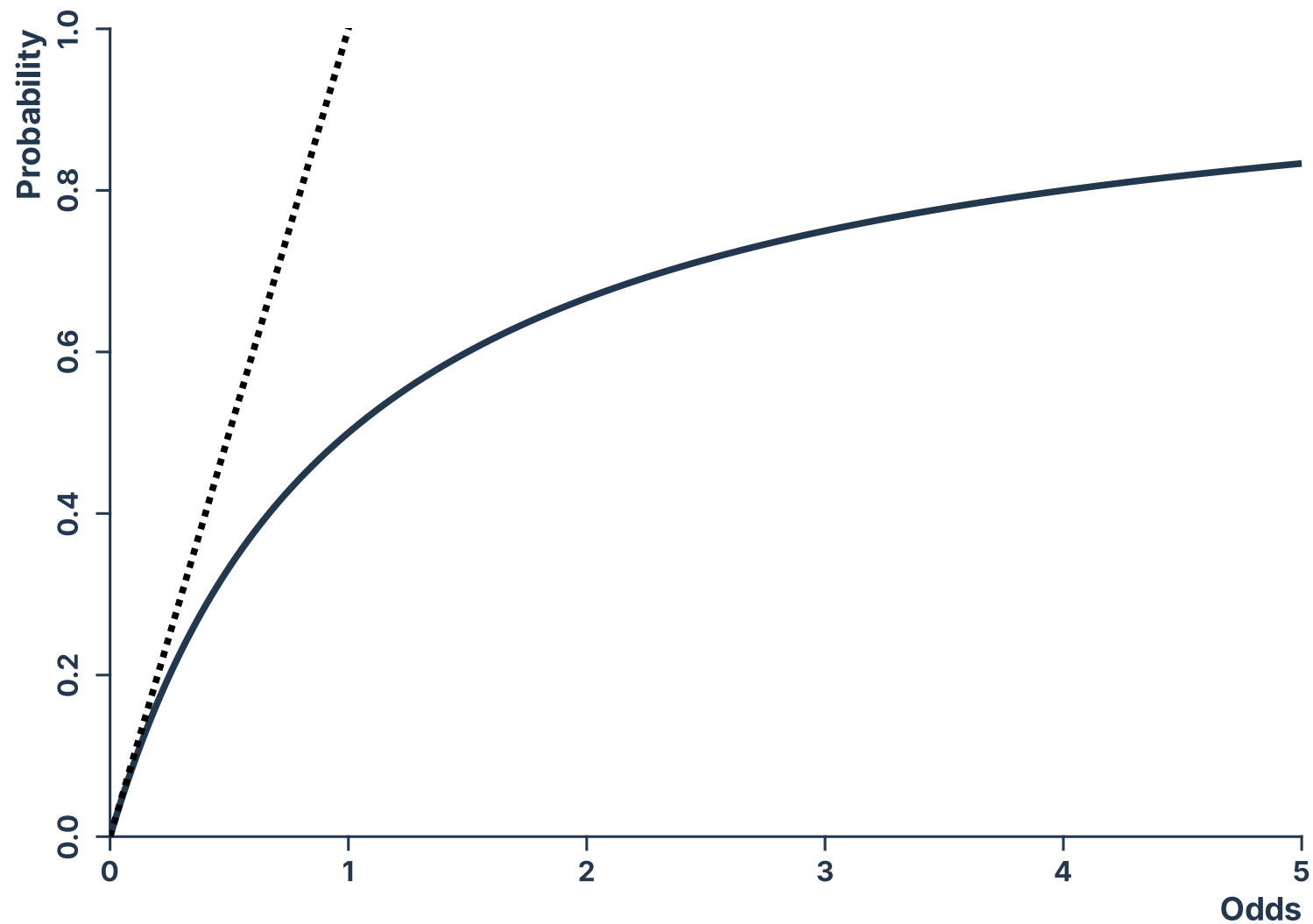
| "For every unit increase in the covariate, the expected **odds** of the outcome is multiplied by $\exp(\beta)$ "

| "For every grade a student completes, the expected **odds** of having tried cocaine is multiplied by 17% (multiplied by 1.17)"

ODDS RATIOS ARE UNINTUITIVE

10

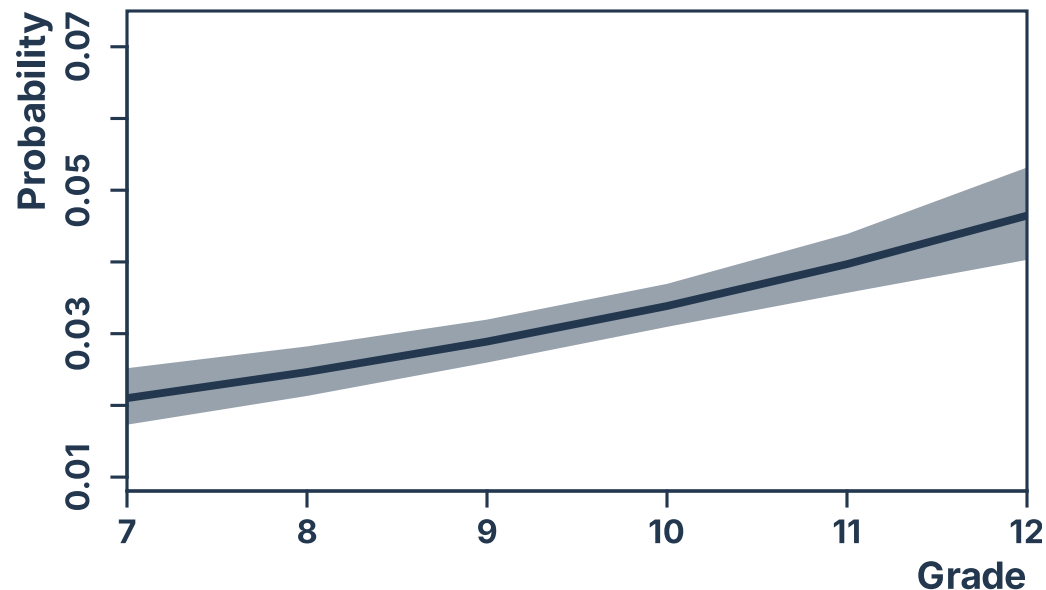
The probability scale of the odds ratio
depends on "initial" probability,



Cases

An average 7th grade student has about a 2.1% (1.6%, 2.8%) chance of having tried cocaine, while for an average 12th grader, that probability is about 4.7% (3.7%, 5.7%)

Posterior visualization



Average marginal effects

Among the students in our sample, completing a grade is predicted to increase the probability of having tried cocaine by about 17.3% (17.1%, 17.4%) on average

Multiplicative on posterior probability

$$C_i \sim \text{Bernoulli}(p_i)$$

$$\text{logit}(p_i) = \alpha + \beta_G G_i + \beta_D D_i + \beta_W W_i$$

$$\alpha \sim \text{Norm}(0, 1.7)$$

$$\beta_G, \beta_D, \beta_W \sim \text{Norm}(0, 0.4)$$

G_i : Grade in school
(centered at grade 9)

D_i : Delinquency
(standardized)

W_i : White (indicator)

	Mean	95% C.I.
α	-4.48	(-4.81, -4.16)
β_G	0.22	(0.13, 0.32)
β_D	0.89	(0.78, 1.00)
β_W	0.62	(0.32, 0.94)