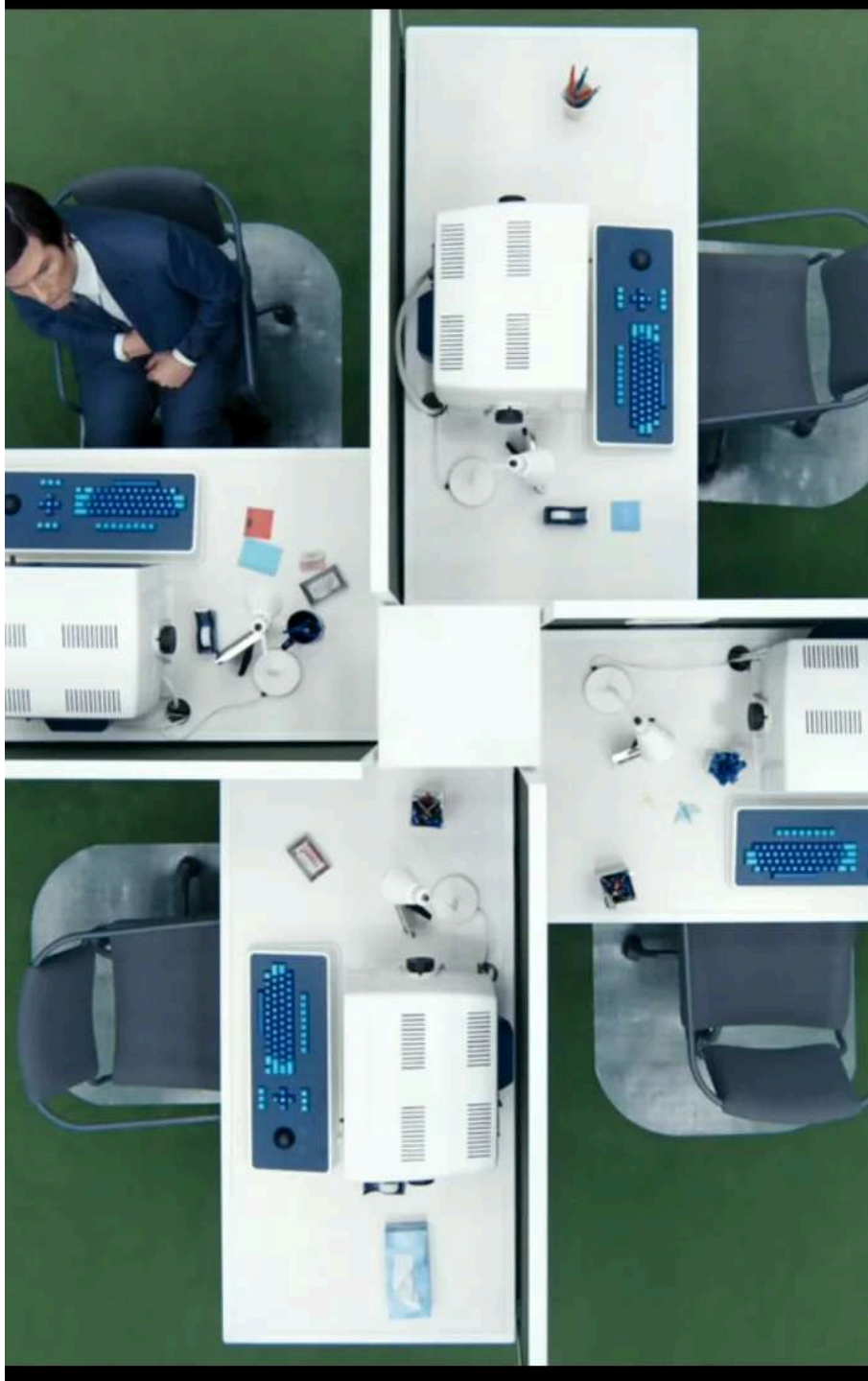
## Agenda

Covariates for causal analysis

1. Administrative
2. Interacting variables in regression
3. Causal analysis in regression
4. Mediation, moderation, confounding, and collision
5. ***Hands on***: *Building indicator variables in R*

# Predicting income: *interactions*



# DUMMY VARIABLES



$$\log(Inc_i) \sim \text{Norm}(\mu_i, \sigma)$$
$$\mu_i = \alpha + \beta_1 M_i + \beta_2 A_i$$
$$\alpha, \beta_1, \beta_2 \sim \text{Norm}(0, 30)$$
$$\sigma \sim \text{Unif}(0, 50)$$

$M_i$
Indicator variable for male

$A_i$
Indicator variable for respondents at least 35 years old

| | Mean | Std. dev | 2.5% | 97.5% |
|---|---|---|---|---|
| $\alpha$ | 10.16 | 0.03 | 10.10 | 10.21 |
| $\beta_1$ | 0.13 | 0.03 | 0.07 | 0.19 |
| $\beta_2$ | 0.59 | 0.03 | 0.54 | 0.65 |
| $\sigma$ | 0.81 | 0.01 | 0.79 | 0.83 |

$\beta_1$: $\exp(0.13) \approx 1.14$
(men make about 14% more than women, on average)

$\beta_2$: $\exp(0.59) \approx 1.81$
(people at least 35 years old make about 81% more than those under 35, on average)

# INTERACTING DUMMIES



$$\log(Inc_i) \sim \text{Norm}(\mu_i, \sigma)$$

$$\mu_i = \alpha + \beta_1 M_i + \beta_2 A_i + \beta_3 M_i A_i$$

$$\alpha, \beta_1, \beta_2 \sim \text{Norm}(0, 30)$$

$$\sigma \sim \text{Unif}(0, 50)$$

$M_i A_i$
*Interaction* between both indicators

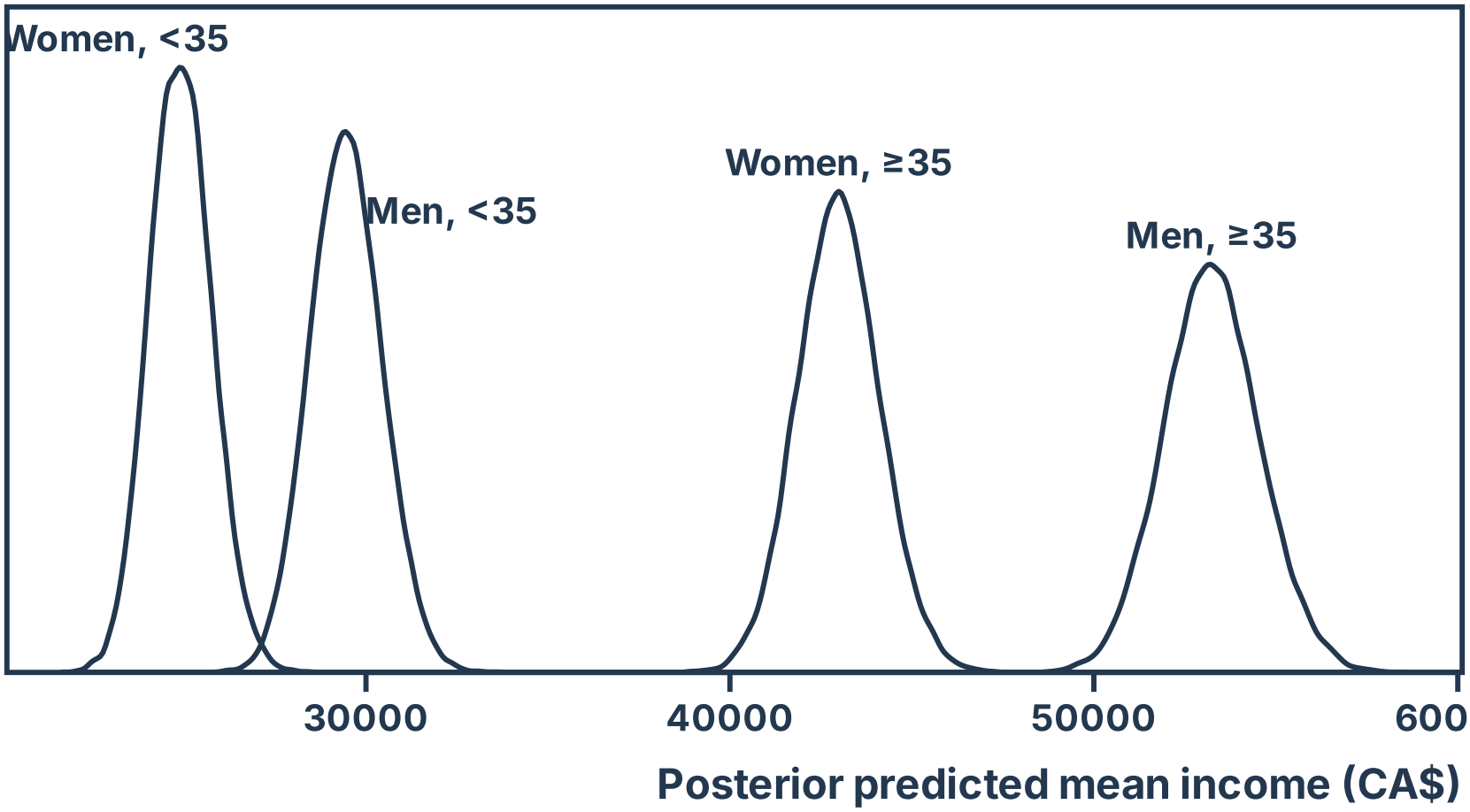
| | Mean | Std. dev | 2.5% | 97.5% |
|---|---|---|---|---|
| $\alpha$ | 10.20 | 0.03 | 10.13 | 10.27 |
| $\beta_1$ | 0.05 | 0.05 | -0.04 | 0.14 |
| $\beta_2$ | 0.52 | 0.04 | 0.44 | 0.61 |
| $\beta_3$ | 0.13 | 0.06 | 0.02 | 0.25 |
| $\sigma$ | 0.81 | 0.01 | 0.79 | 0.83 |

# INTERACTING DUMMIES



$$\mu_i = \alpha + \beta_1 M_i + \beta_2 A_i + \beta_3 M_i A_i$$

$$\mu_{(<35,female)} = \alpha$$
$$\mu_{(<35,male)} = \alpha + \beta_1$$
$$\mu_{(\geq 35,female)} = \alpha + \beta_2$$
$$\mu_{(\geq 35,male)} = \alpha + \beta_1 + \beta_2 + \beta_3$$



# INTERACTING DUMMIES

$$\mu_i = \alpha + \beta_1 M_i + \beta_2 A_i + \beta_3 M_i A_i$$

$$\begin{aligned}
\mu_{(<35, female)} &= \alpha \\
\mu_{(<35, male)} &= \alpha + \beta_1 \\
\mu_{(\geq 35, female)} &= \alpha + \beta_2 \\
\mu_{(\geq 35, male)} &= \alpha + \beta_1 + \beta_2 + \beta_3
\end{aligned}$$

| | Mean | Exp(Mean) |
|---|---|---|
| $\alpha$ | 10.20 | 26,853 |
| $\beta_1$ | 0.05 | 1.05 |
| $\beta_2$ | 0.52 | 1.69 |
| $\beta_3$ | 0.13 | 1.14 |

## Interpreting the interaction coefficient $\beta_3$

The pay boost for being over 35 years old ($\beta_2$) is about 14% greater for men than for women.

***OR***

The income advantage for men over women ($\beta_1$) is about 14% greater for respondents over 35 years old.

# INTERACTING CONTINUOUS VARIABLES

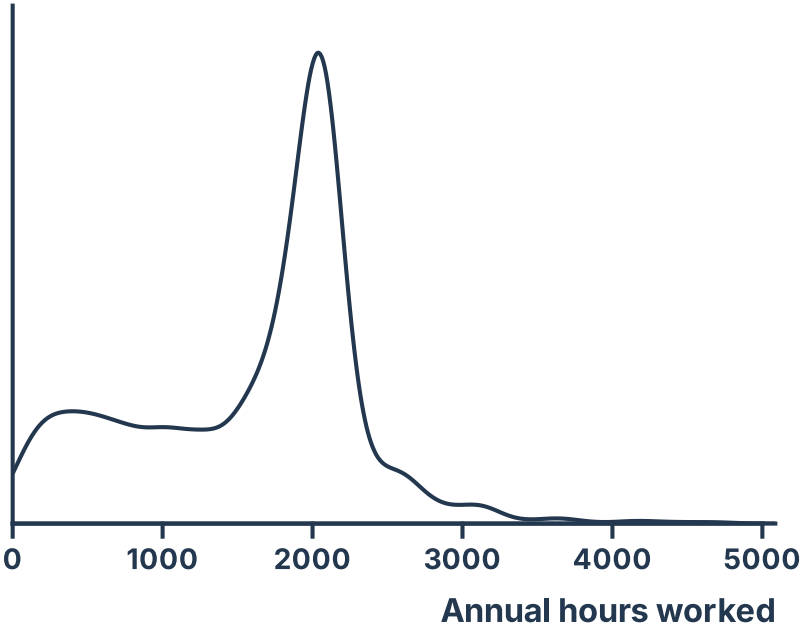
$$\log(Inc_i) \sim \text{Norm}(\mu_i, \sigma)$$

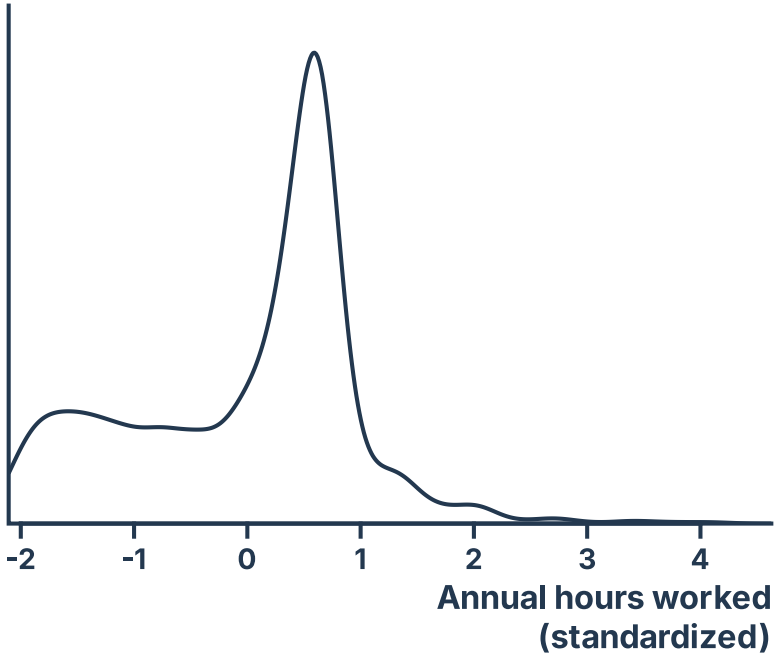
$$\mu_i = \alpha + \beta_1 Hours_i + \beta_2 Dues_i + \beta_3 Hours_i Dues_i$$

Hours worked (standardized)

Union and prof. assoc. dues (standardized)

**Standardization:** Transforming a variable $X$ so that $\text{mean}(X) = 0$ and $\text{sd}(X) = 1$



# INTERACTING CONTINUOUS VARIABLES



$$\mu_i = \alpha + \beta_1 Hours_i + \beta_2 Dues_i + \beta_3 Hours_i Dues_i$$

| | Mean | Exp(Mean) |
|---|---|---|
| $\alpha$ | 10.58 | 39,549 |
| $\beta_1$ | 0.40 | 1.49 |
| $\beta_2$ | 0.20 | 1.22 |
| $\beta_3$ | -0.07 | 0.93 |

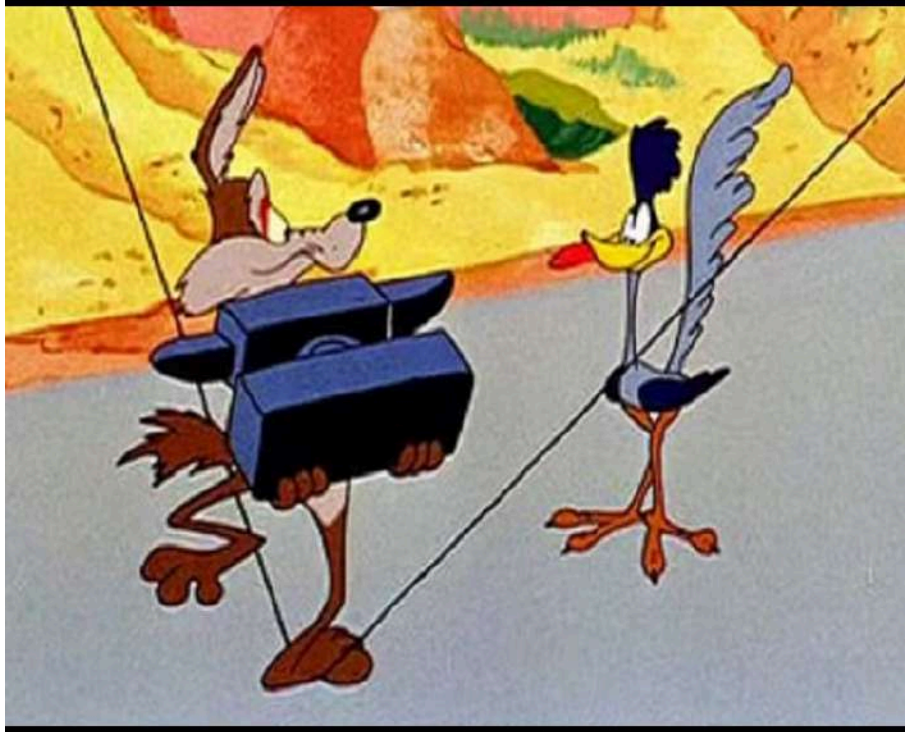
## Interpreting the interaction coefficient $\beta_3$

The extra income associated with increased union and professional dues ($\beta_2$) is reduced by about 7% for every standard-deviation increase in hours worked.

***OR***

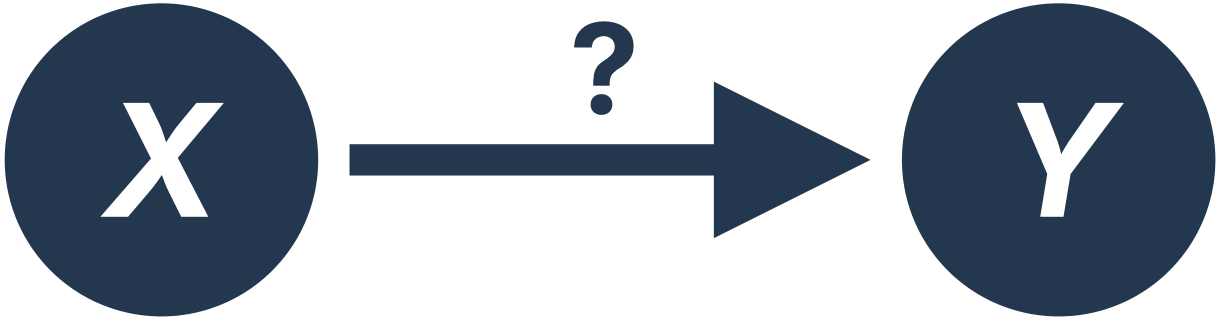
The extra money made by working more hours ($\beta_1$) is reduced by about 7% for every standard-deviation increase in union dues paid.

# Identifying cause & effect



# CAUSAL ANALYSIS






**Causal question:** Does a change in one variable (*X*) ***cause*** a change in another (*Y*)?

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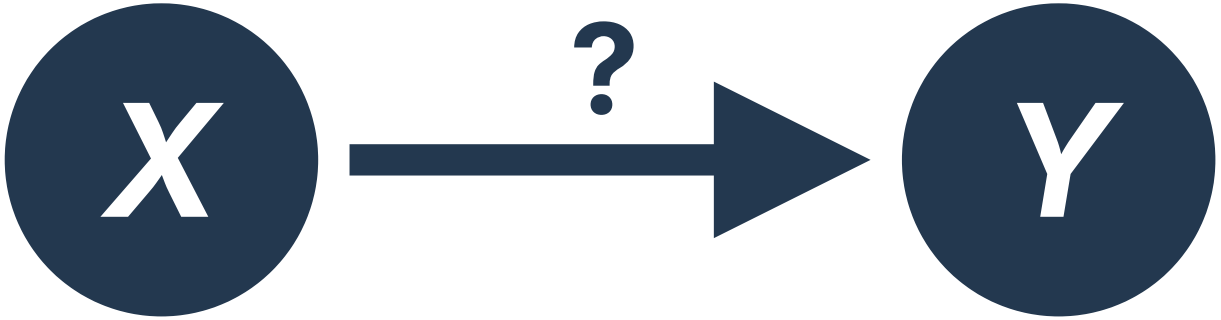
*Regression* only identifies statistical relationships, not causal relationships

To draw a "causal arrow" you need ***theory***

# CAUSAL ANALYSIS



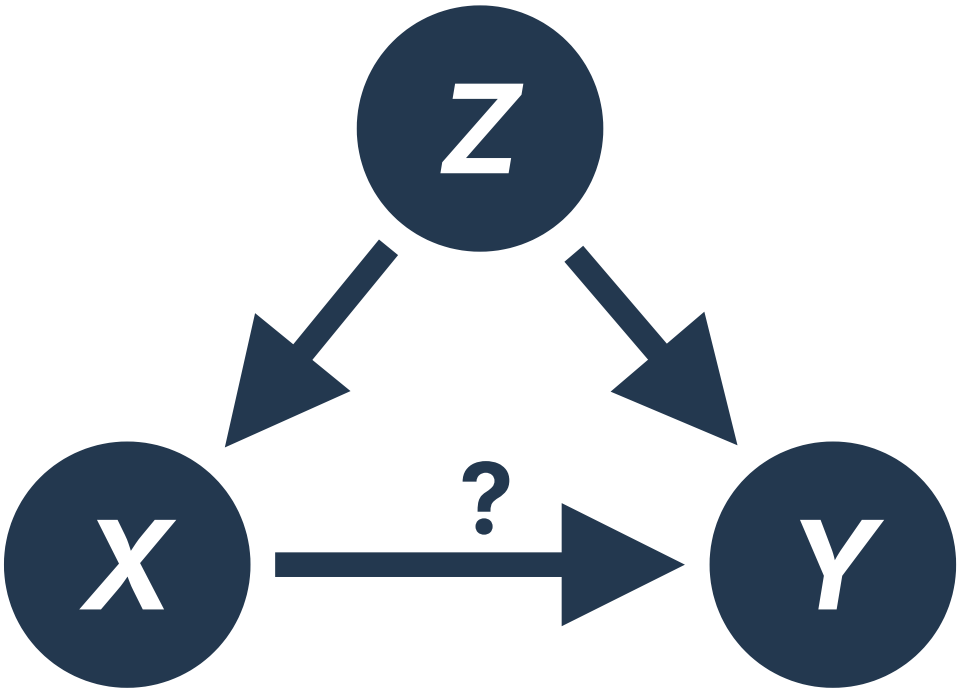



**To establish a causal relationship you (usually) need**

1. **Causal precedence**
   A theoretical reason to believe that changes in *X* *could* affect *Y* (e.g. precedes *Y* in time)
2. **Statistical association**
   An established statistical association between *X* and *Y* (e.g. a convincing coefficient estimate)
3. **No unaccounted-for confounders**
   No other variables, observed or otherwise, that *confound* the association between *X* and *Y*

# CONFOUNDING VARIABLES

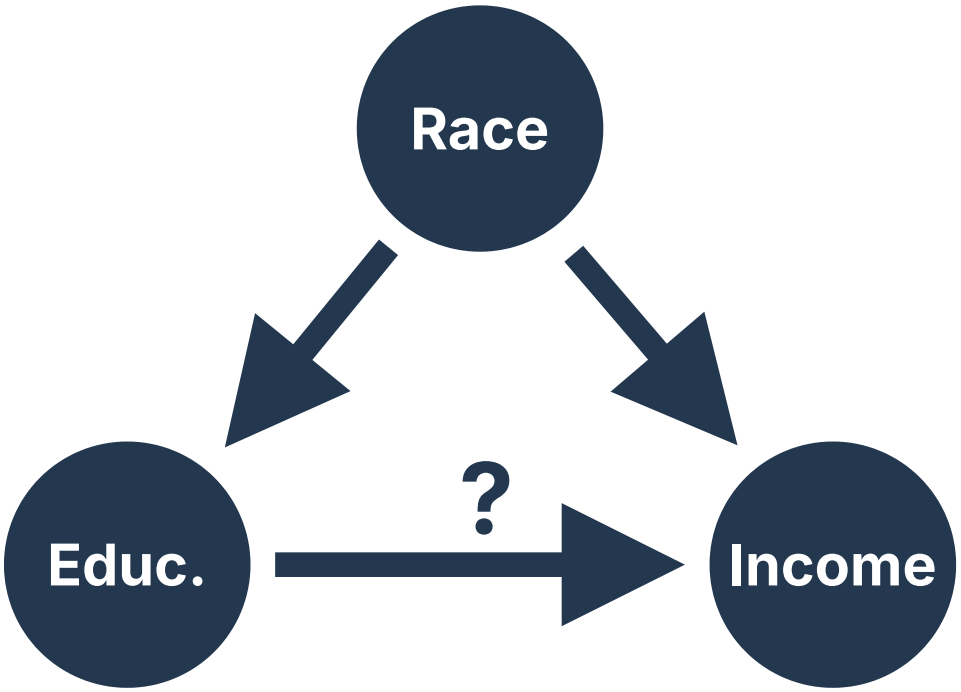





A variable *Z* is a ***confounder*** of the relationship between *X* and *Y* if *Z* is a causal influence on both *X* and *Y*

# CONFOUNDING VARIABLES





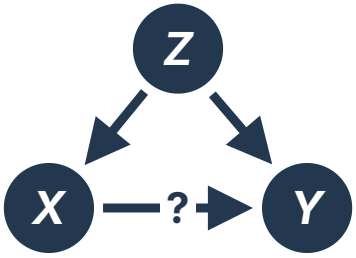

A variable *Z* is a ***confounder*** of the relationship between *X* and *Y* if *Z* is a causal influence on both *X* and *Y*

**For example:**
To establish a causal relationship between education and income, you need to account for race, which could affect both education and income

# TYPES OF COVARIATES



## Confounder

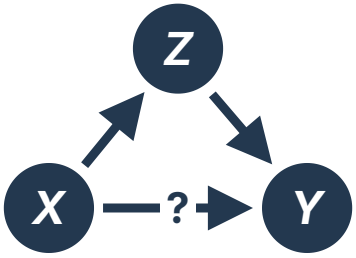



*Z* is a causal factor on both *X* and *Y*.

Must be "controlled for" to establish non-spurious relationship between *X* and *Y*.

*E.g.:*
*Race confounds the relationship between education and income.*

## Mediator

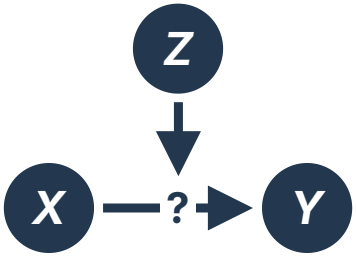



*Z* is influenced by *X* and influences *Y*.

Including as covariate elaborates on relationship between *X* and *Y*.

*E.g.:*
*Occupation mediates the relationship between gender and income.*

## Moderator

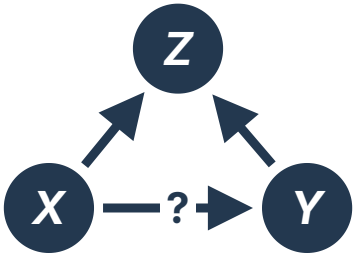



*Z* alters the relationship between *X* and *Y*.

Can be included as interaction variable to better describe the relationship between *X* and *Y*.

*E.g.:*
*Marital status moderates the relationship between gender and income.*

## Collider



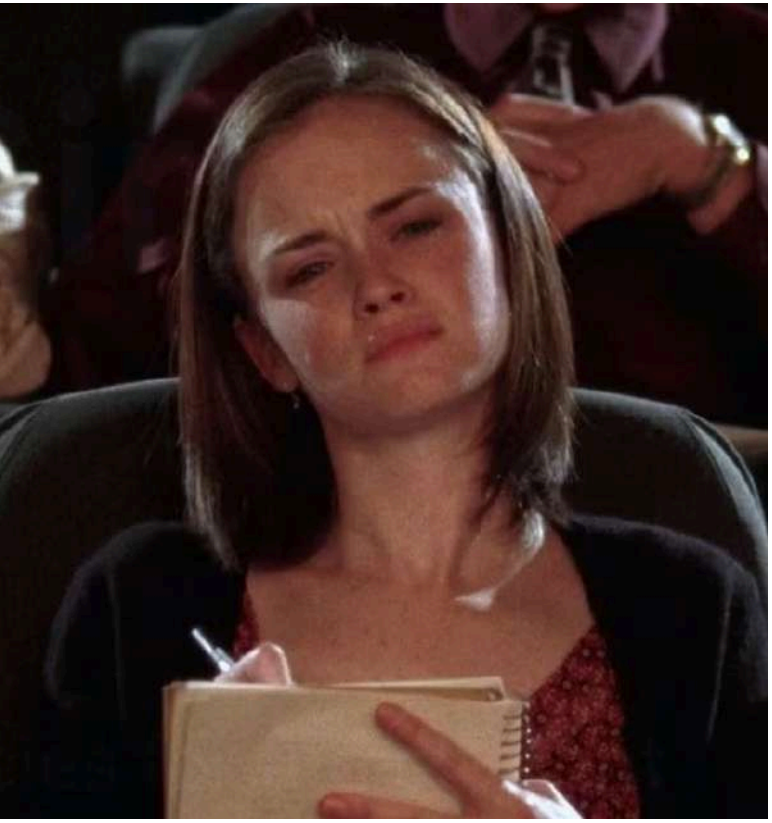

*Z* causally influenced by both *X* and *Y*.

Must ***not*** be "controlled for" when establishing relationship between *X* and *Y*.

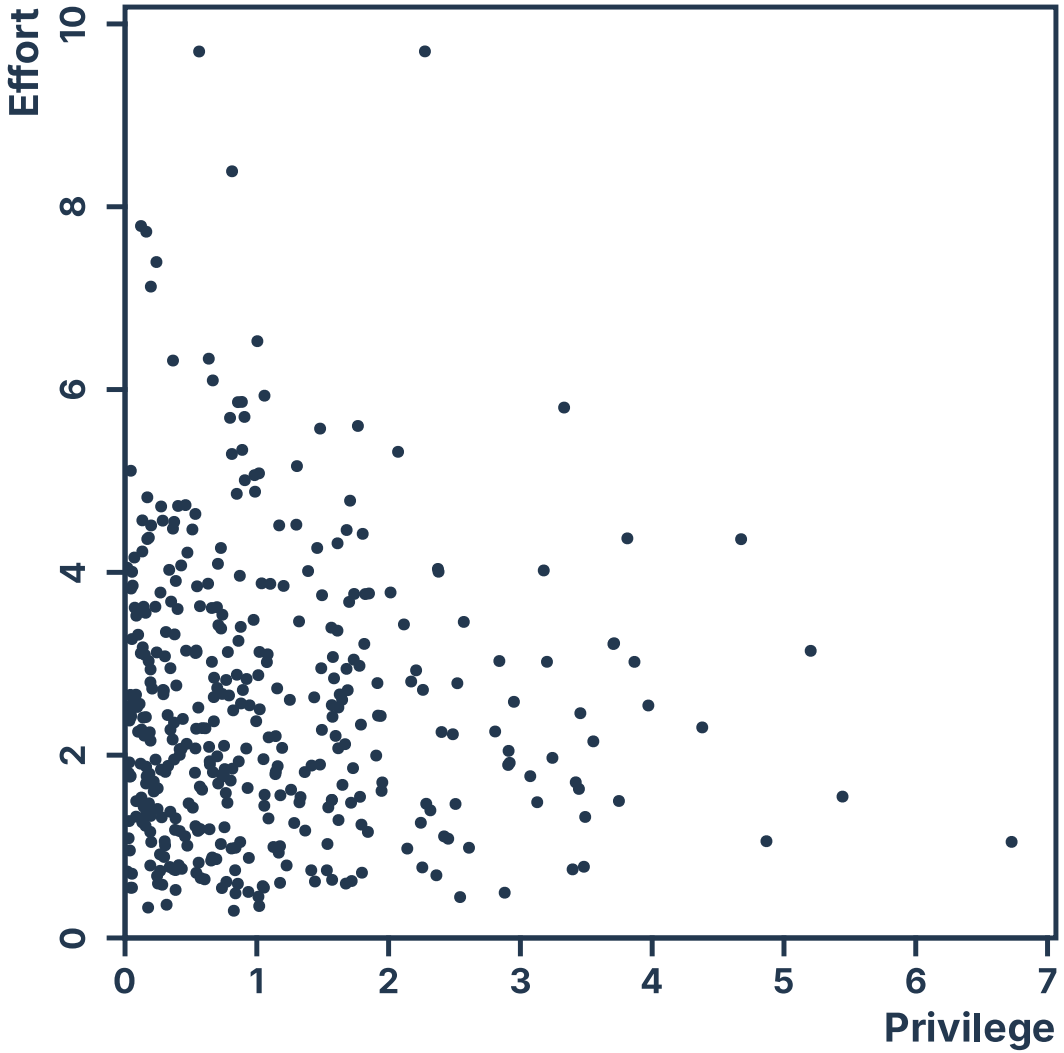
*E.g.:*
*Income is a collider for the relationship between gender and occupation.*

# COLLIDER BIAS: AN EXAMPLE





Do college students who come from privileged backgrounds dedicate less effort to studying?



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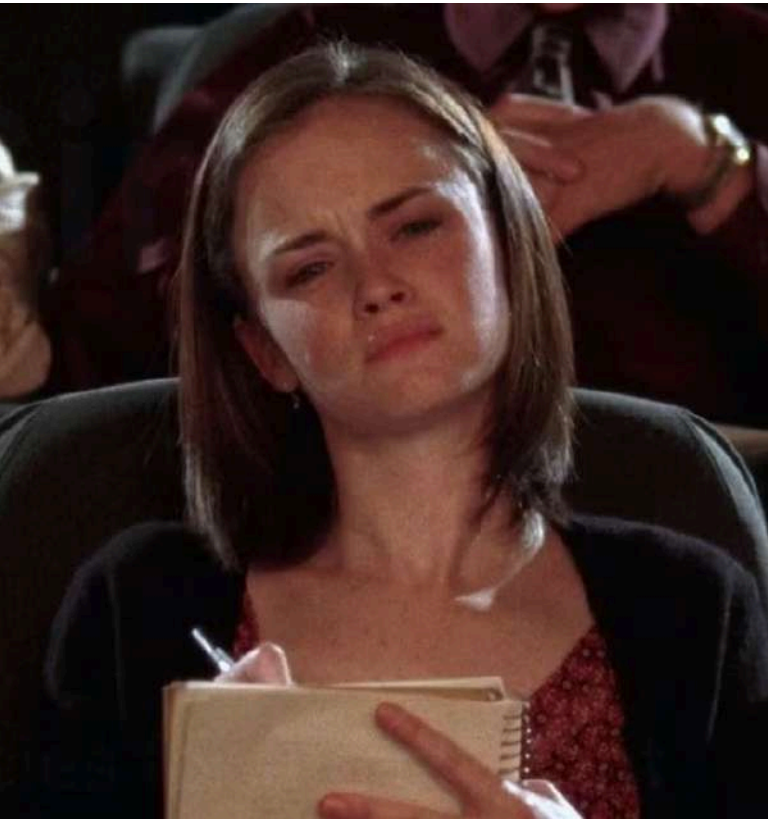

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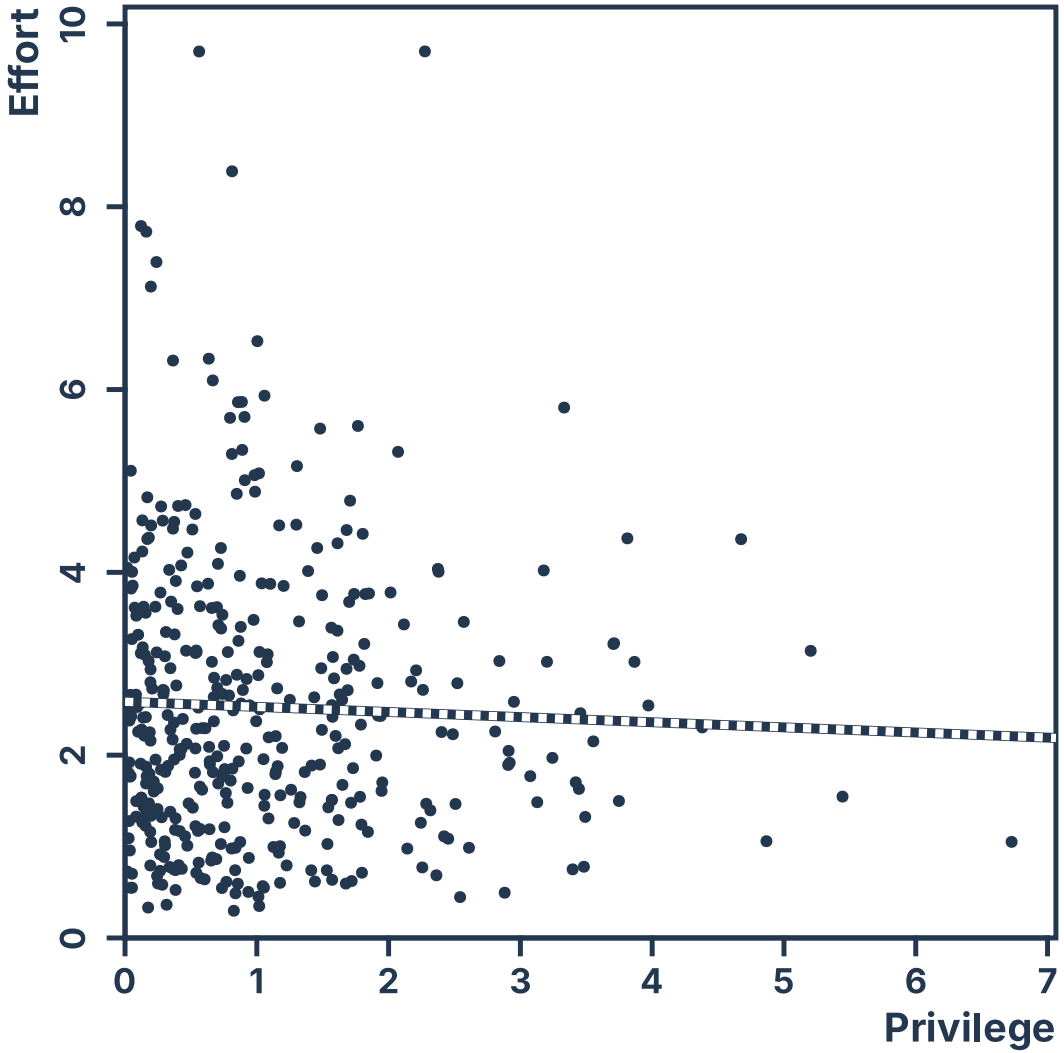

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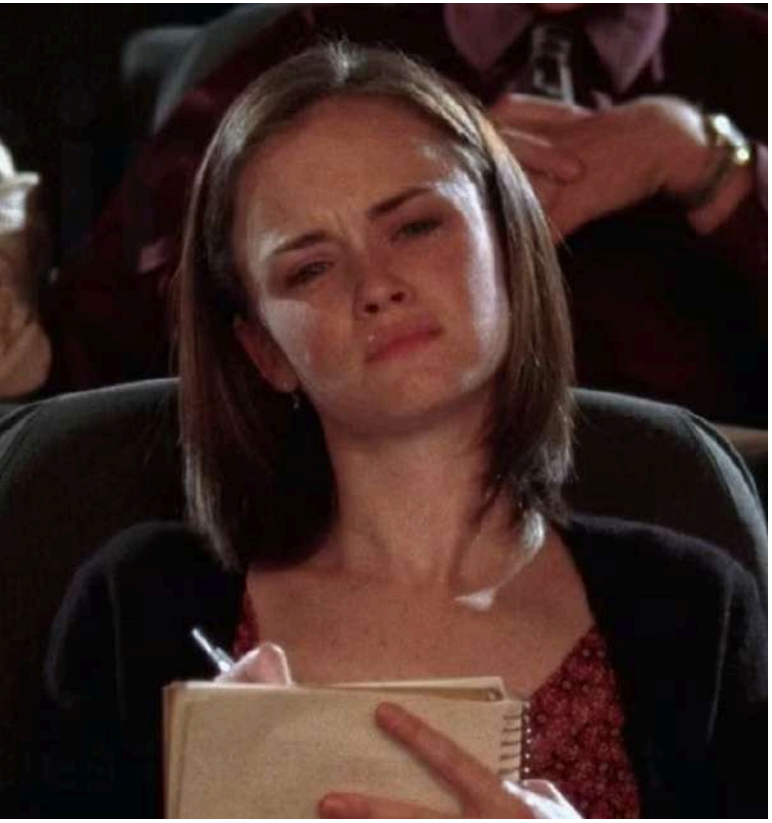

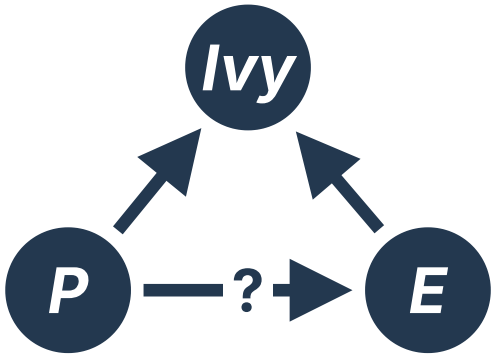


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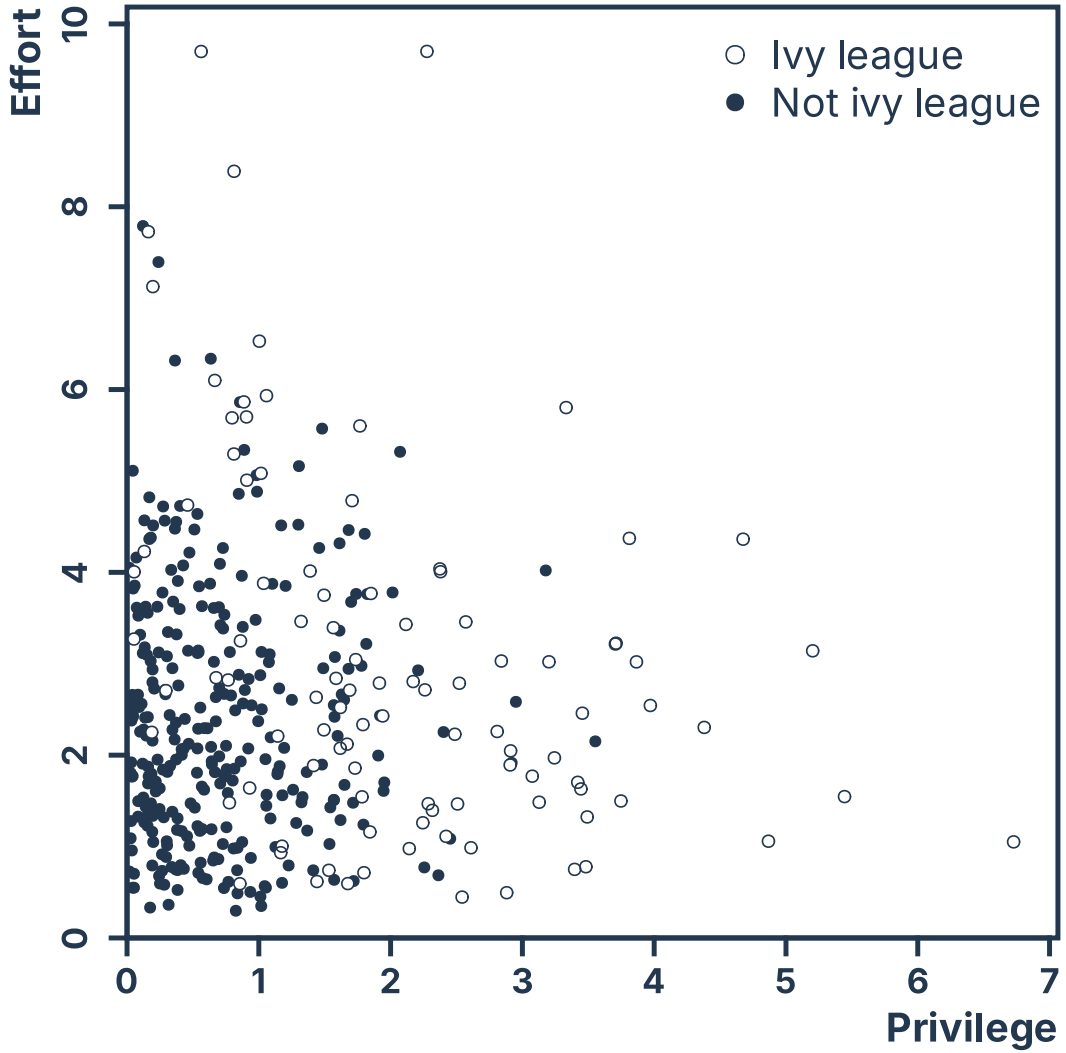

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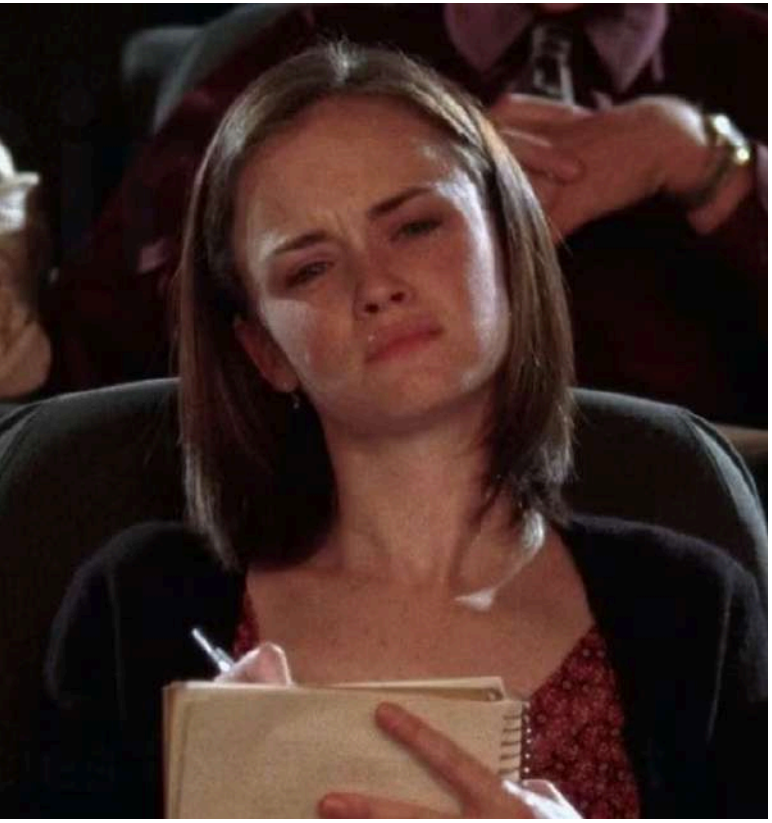

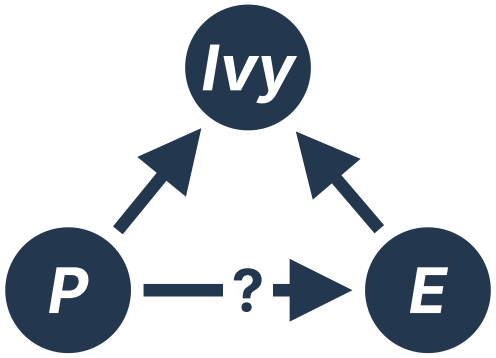


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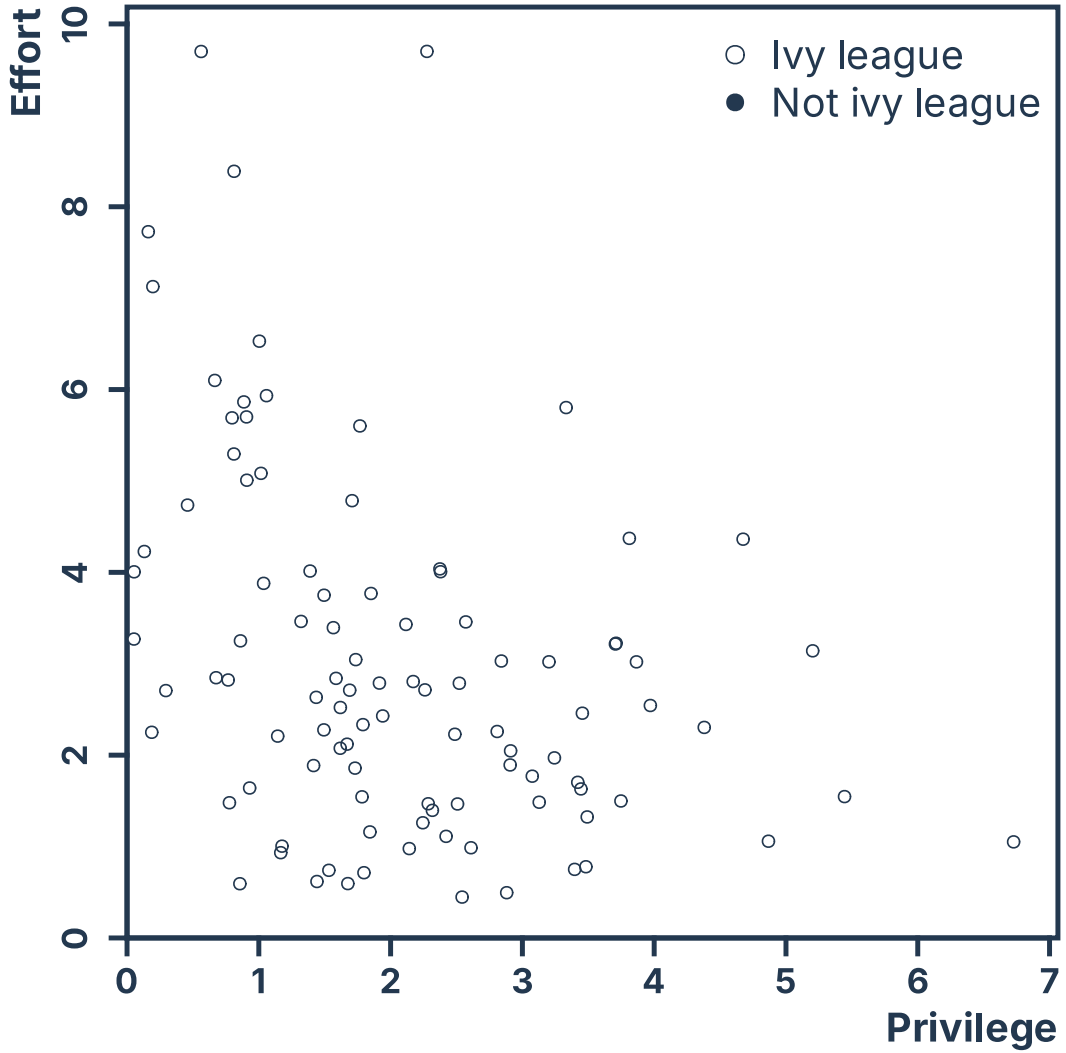

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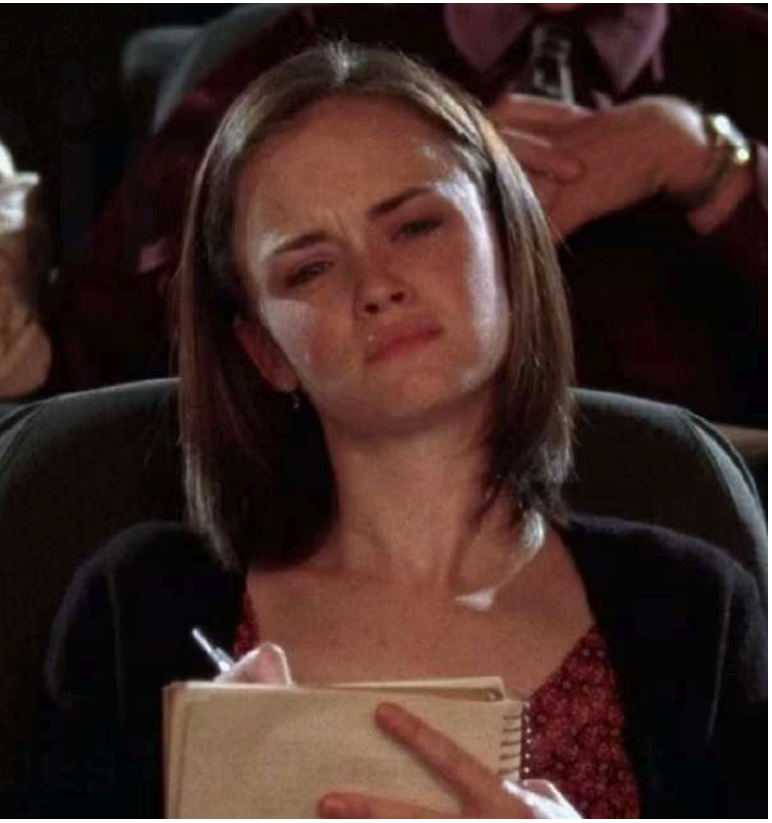

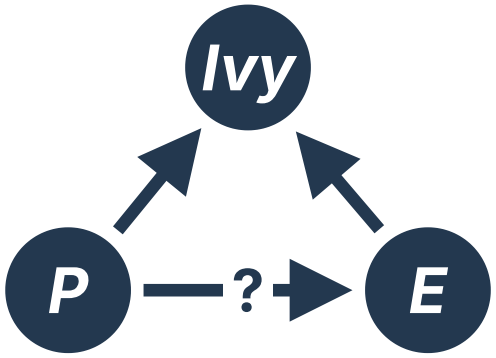


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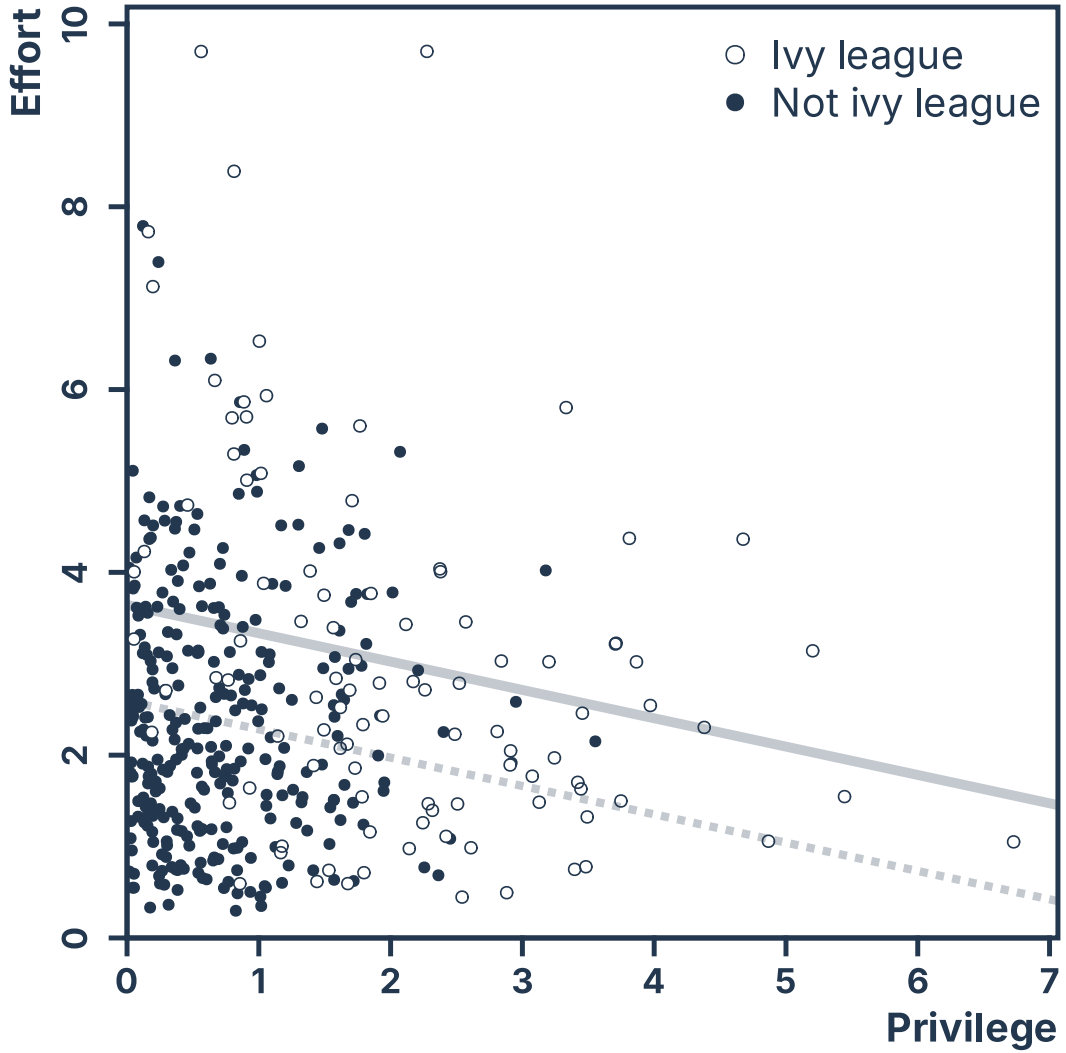
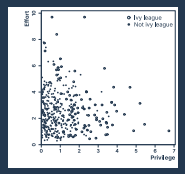
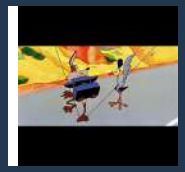


# Image credit

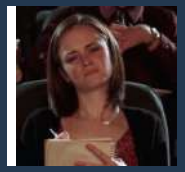



Figures by Peter McMahan (source code)

Still from Severance (2022)



Road Runner and Wile E. Coyote © Warner Bros. Entertainment



Still from Gilmore Girls (2000)