Agenda

Covariates for causal analysis

1. Administrative
2. Interacting variables in regression
3. Causal analysis in regression
4. Mediation, moderation, confounding, and collision
5. ***Hands on***: *Building indicator variables in R*

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Updated worksheet dates

- First worksheet due *Tuesday*, Sept 22 by midnight
- Peer assessments due that Friday, Sept 25

Predicting income: *interactions*

DUMMY VARIABLES

$$\log(Inc_i) \sim \text{Norm}(\mu_i, \sigma)$$
$$\mu_i = \alpha + \beta_1 M_i + \beta_2 A_i$$
$$\alpha, \beta_1, \beta_2 \sim \text{Norm}(0, 30)$$
$$\sigma \sim \text{Unif}(0, 50)$$

M_i
Indicator variable for male

A_i
Indicator variable for respondents at least 35 years old

	Mean	Std. dev	2.5%	97.5%
α	10.16	0.03	10.10	10.21
β_1	0.13	0.03	0.07	0.19
β_2	0.59	0.03	0.54	0.65
σ	0.81	0.01	0.79	0.83

β_1: $\exp(0.13) \approx 1.14$
(men make about 14% more than women, on average)

β_2: $\exp(0.59) \approx 1.81$
(people at least 35 years old make about 81% more than those under 35, on average)

INTERACTING DUMMIES

$$\log(Inc_i) \sim \text{Norm}(\mu_i, \sigma)$$

$$\mu_i = \alpha + \beta_1 M_i + \beta_2 A_i + \beta_3 M_i A_i$$

$$\alpha, \beta_1, \beta_2 \sim \text{Norm}(0, 30)$$

$$\sigma \sim \text{Unif}(0, 50)$$

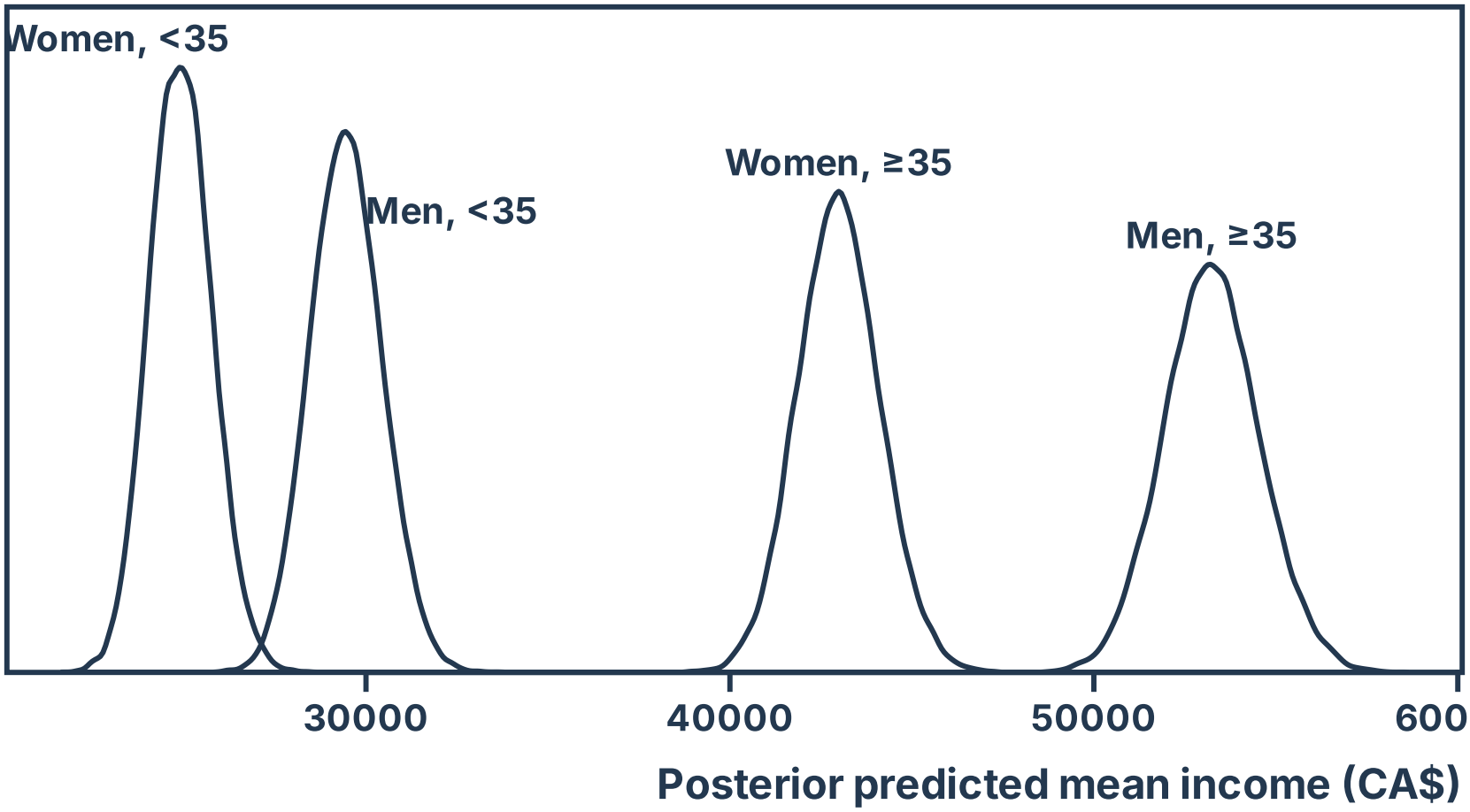
$M_i A_i$
Interaction between both indicators

	Mean	Std. dev	2.5%	97.5%
α	10.20	0.03	10.13	10.27
β_1	0.05	0.05	-0.04	0.14
β_2	0.52	0.04	0.44	0.61
β_3	0.13	0.06	0.02	0.25
σ	0.81	0.01	0.79	0.83

INTERACTING DUMMIES

$$\mu_i = \alpha + \beta_1 M_i + \beta_2 A_i + \beta_3 M_i A_i$$

$$\mu_{(<35,female)} = \alpha$$
$$\mu_{(<35,male)} = \alpha + \beta_1$$
$$\mu_{(\geq 35,female)} = \alpha + \beta_2$$
$$\mu_{(\geq 35,male)} = \alpha + \beta_1 + \beta_2 + \beta_3$$



INTERACTING DUMMIES

$$\mu_i = \alpha + \beta_1 M_i + \beta_2 A_i + \beta_3 M_i A_i$$

$$\begin{aligned}
\mu_{(<35, female)} &= \alpha \\
\mu_{(<35, male)} &= \alpha + \beta_1 \\
\mu_{(\geq 35, female)} &= \alpha + \beta_2 \\
\mu_{(\geq 35, male)} &= \alpha + \beta_1 + \beta_2 + \beta_3
\end{aligned}$$

	Mean	Exp(Mean)
α	10.20	26,853
β_1	0.05	1.05
β_2	0.52	1.69
β_3	0.13	1.14

Interpreting the interaction coefficient β_3

The pay boost for being over 35 years old (β_2) is about 14% greater for men than for women.

OR

The income advantage for men over women (β_1) is about 14% greater for respondents over 35 years old.

INTERACTING CONTINUOUS VARIABLES

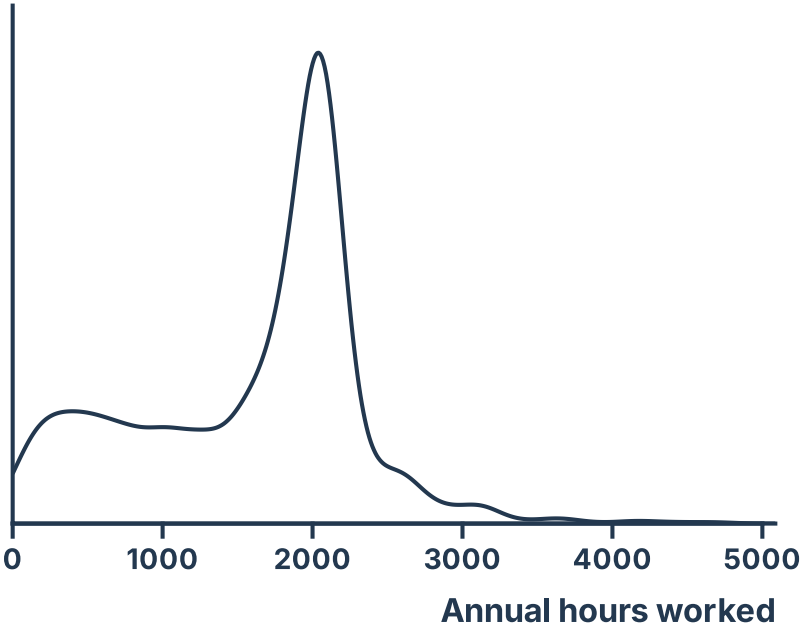
$$\log(Inc_i) \sim \text{Norm}(\mu_i, \sigma)$$

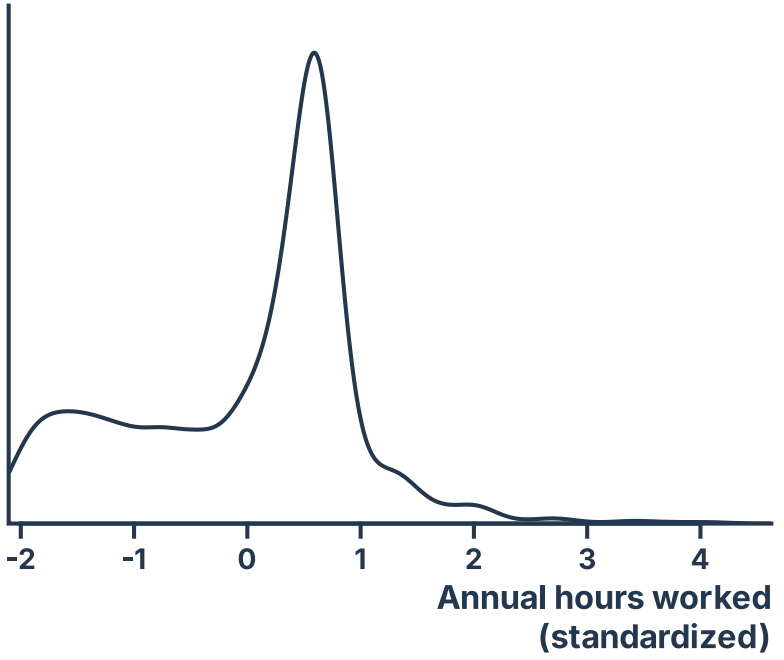
$$\mu_i = \alpha + \beta_1 Hours_i + \beta_2 Dues_i + \beta_3 Hours_i Dues_i$$

Hours worked (standardized)

Union and prof. assoc. dues (standardized)

Standardization: Transforming a variable X so that $\text{mean}(X) = 0$ and $\text{sd}(X) = 1$





$$\mu_i = \alpha + \beta_1 Hours_i + \beta_2 Dues_i + \beta_3 Hours_i Dues_i$$

	Mean	Exp(Mean)
α	10.58	39,549
β_1	0.40	1.49
β_2	0.20	1.22
β_3	-0.07	0.93

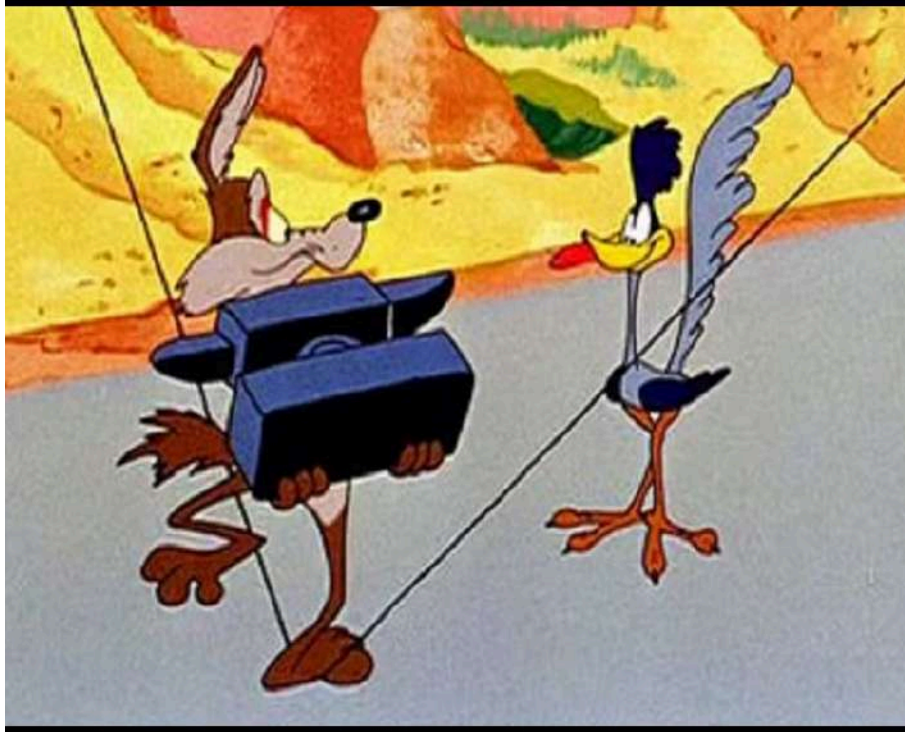
Interpreting the interaction coefficient β_3

The extra income associated with increased union and professional dues (β_2) is reduced by about 7% for every standard-deviation increase in hours worked.

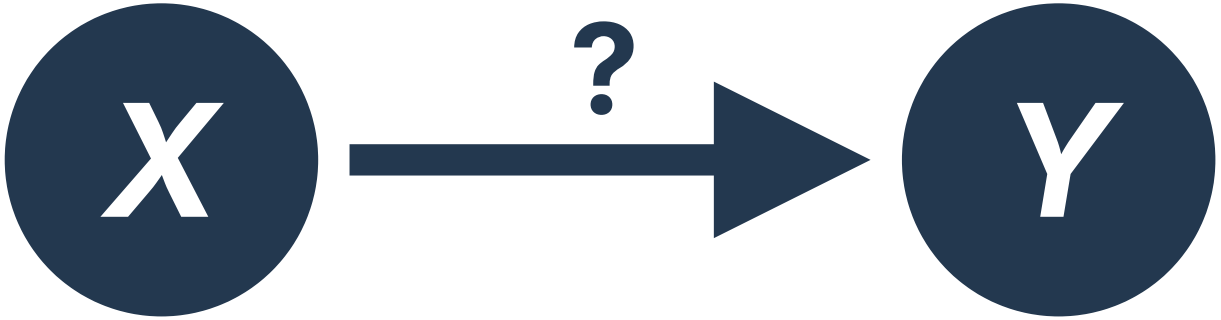
OR

The extra money made by working more hours (β_1) is reduced by about 7% for every standard-deviation increase in union dues paid.

Identifying cause & effect



CAUSAL ANALYSIS

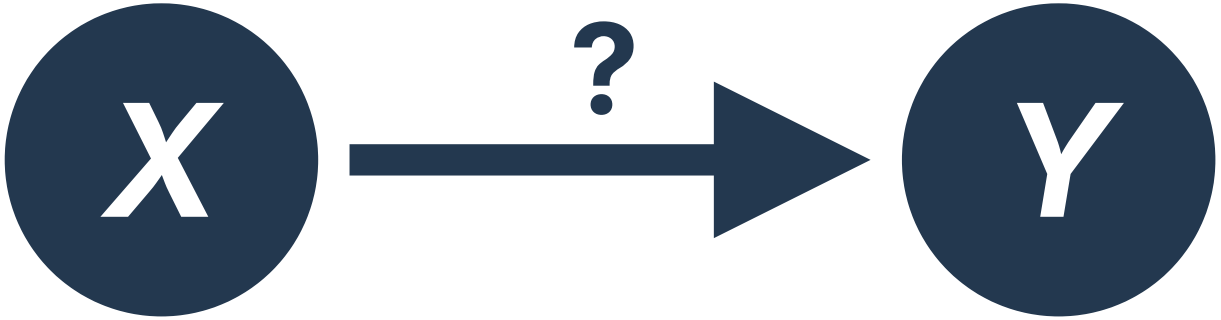


Causal question: Does a change in one variable (*X*) ***cause*** a change in another (*Y*)?

Regression only identifies statistical relationships, not causal relationships

To draw a "causal arrow" you need ***theory***

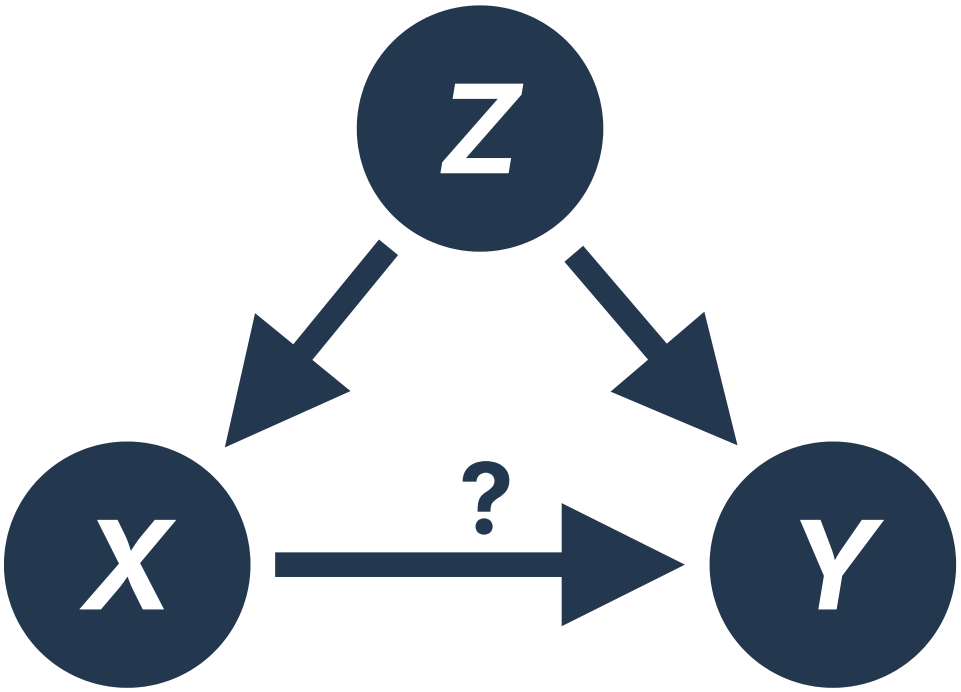
CAUSAL ANALYSIS



To establish a causal relationship you (usually) need

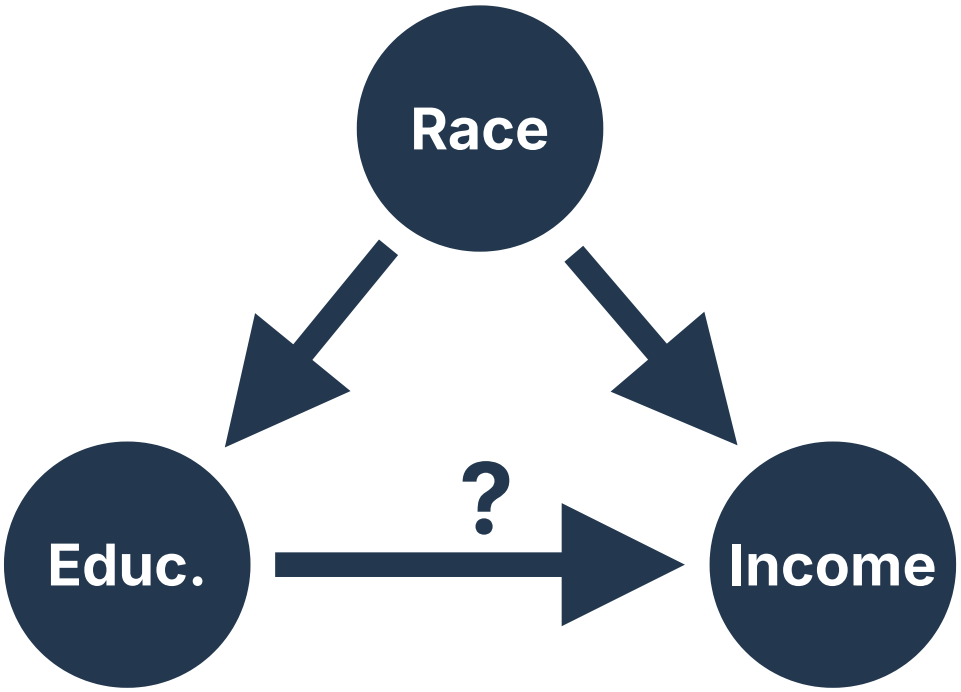
1. **Causal precedence**
   A theoretical reason to believe that changes in *X* *could* affect *Y* (e.g. precedes *Y* in time)
2. **Statistical association**
   An established statistical association between *X* and *Y* (e.g. a convincing coefficient estimate)
3. **No unaccounted-for confounders**
   No other variables, observed or otherwise, that *confound* the association between *X* and *Y*

CONFOUNDING VARIABLES



A variable *Z* is a ***confounder*** of the relationship between *X* and *Y* if *Z* is a causal influence on both *X* and *Y*

CONFOUNDING VARIABLES

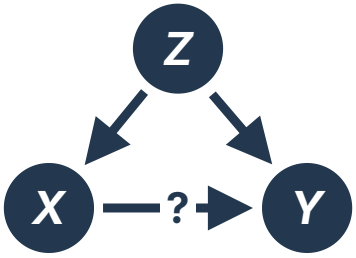


A variable *Z* is a ***confounder*** of the relationship between *X* and *Y* if *Z* is a causal influence on both *X* and *Y*

For example:
To establish a causal relationship between education and income, you need to account for race, which could affect both education and income

TYPES OF COVARIATES

Confounder

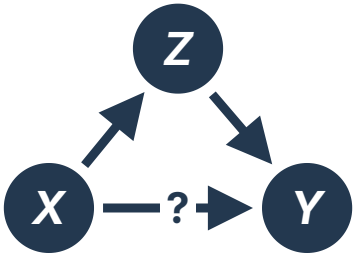


Z is a causal factor on both *X* and *Y*.

Must be "controlled for" to establish non-spurious relationship between *X* and *Y*.

E.g.:
Race confounds the relationship between education and income.

Mediator

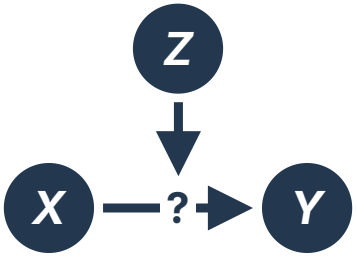


Z is influenced by *X* and influences *Y*.

Including as covariate elaborates on relationship between *X* and *Y*.

E.g.:
Occupation mediates the relationship between gender and income.

Moderator

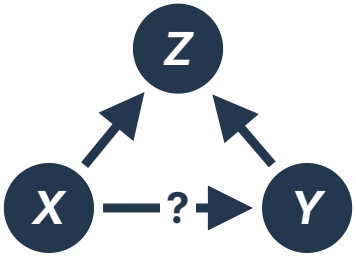


Z alters the relationship between *X* and *Y*.

Can be included as interaction variable to better describe the relationship between *X* and *Y*.

E.g.:
Marital status moderates the relationship between gender and income.

Collider

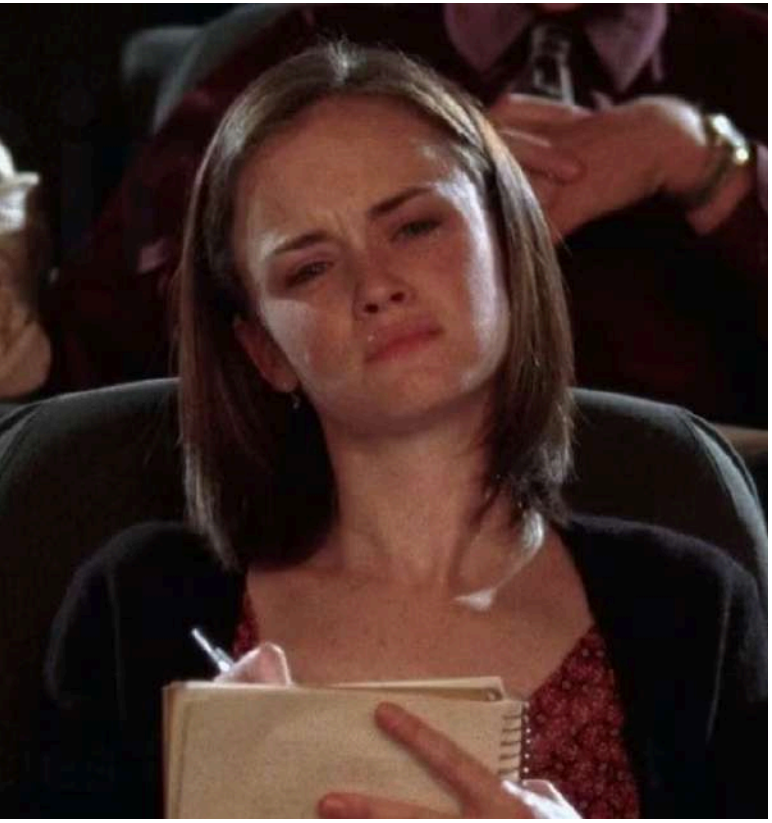


Z causally influenced by both *X* and *Y*.

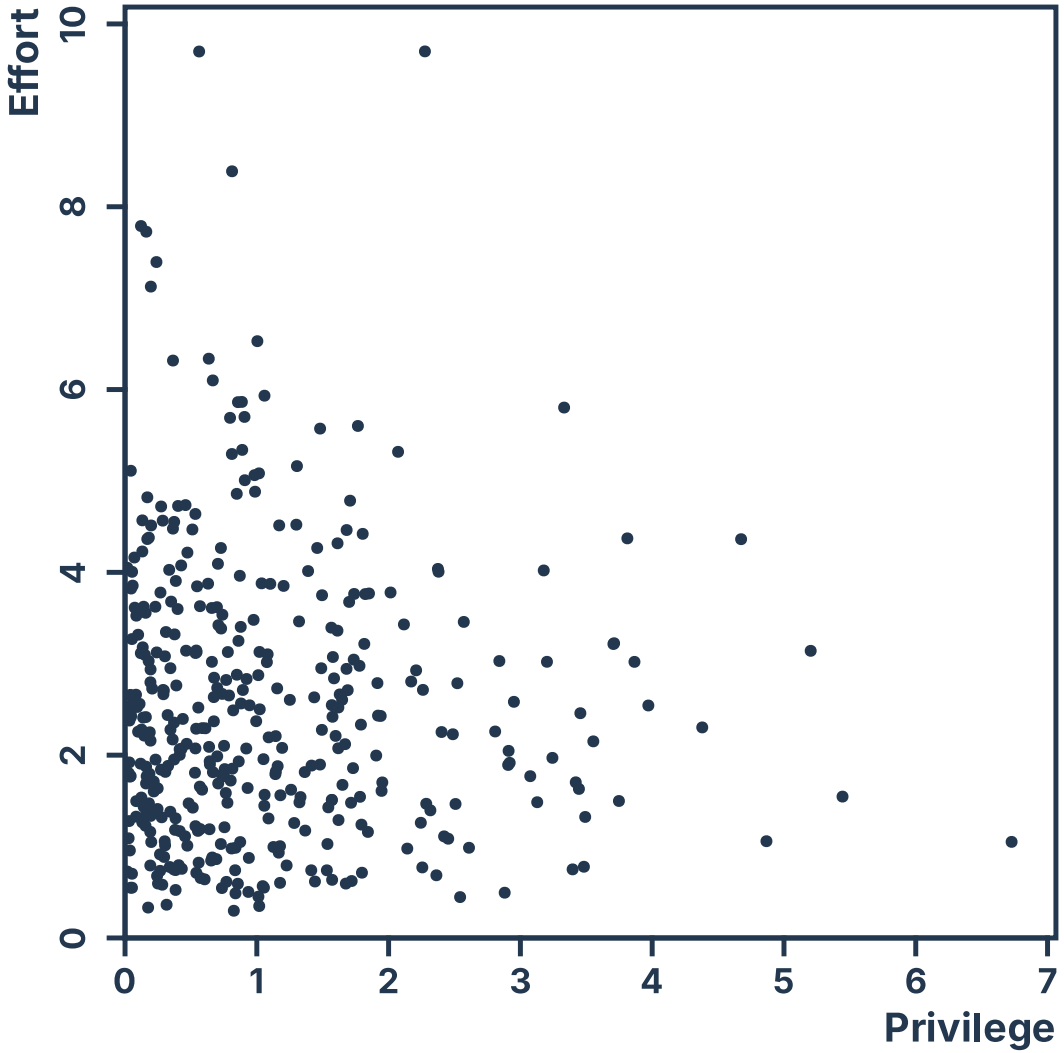
Must ***not*** be "controlled for" when establishing relationship between *X* and *Y*.

E.g.:
Income is a collider for the relationship between gender and occupation.

COLLIDER BIAS: AN EXAMPLE



Do college students who come from privileged backgrounds dedicate less effort to studying?



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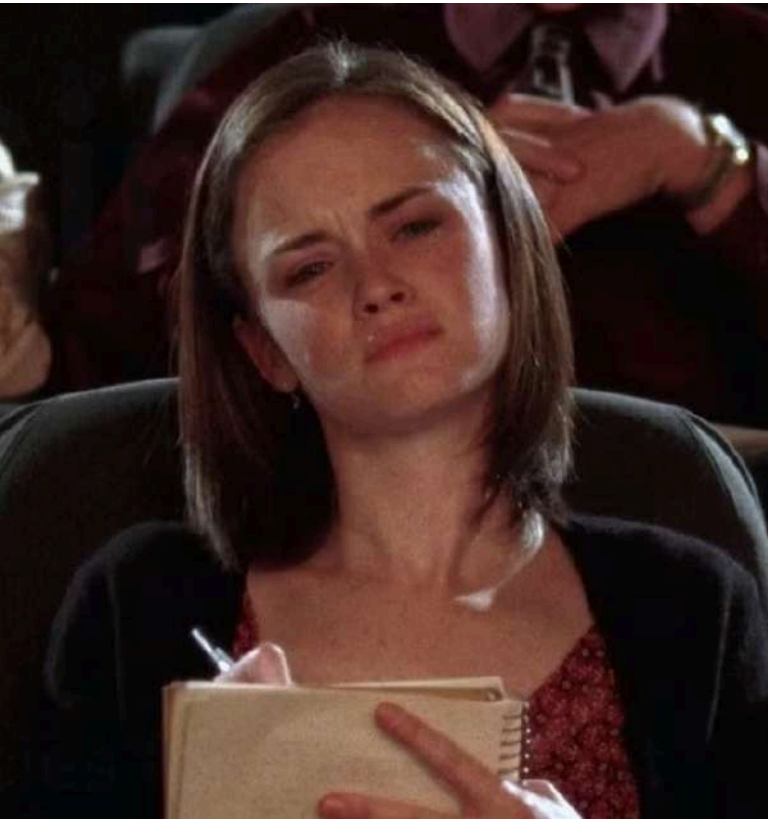

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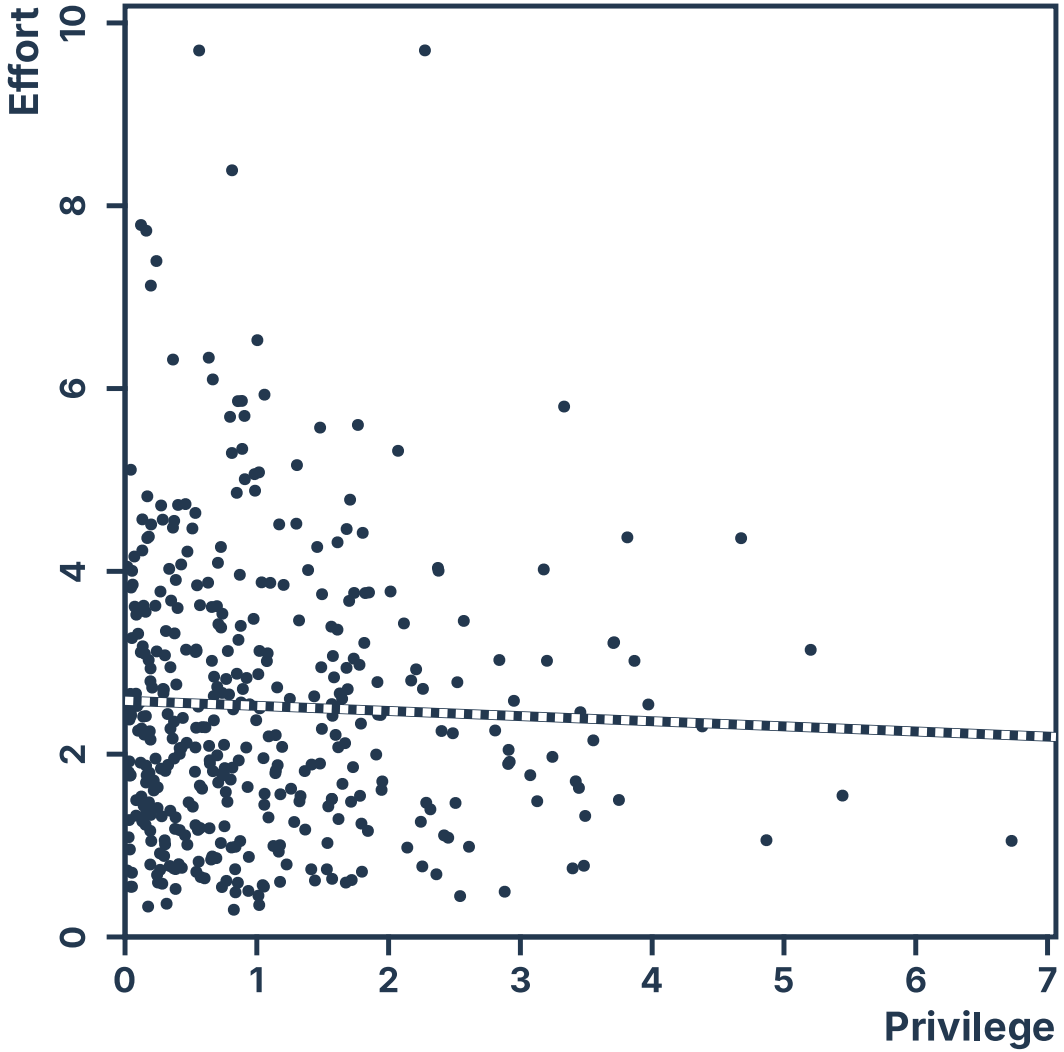

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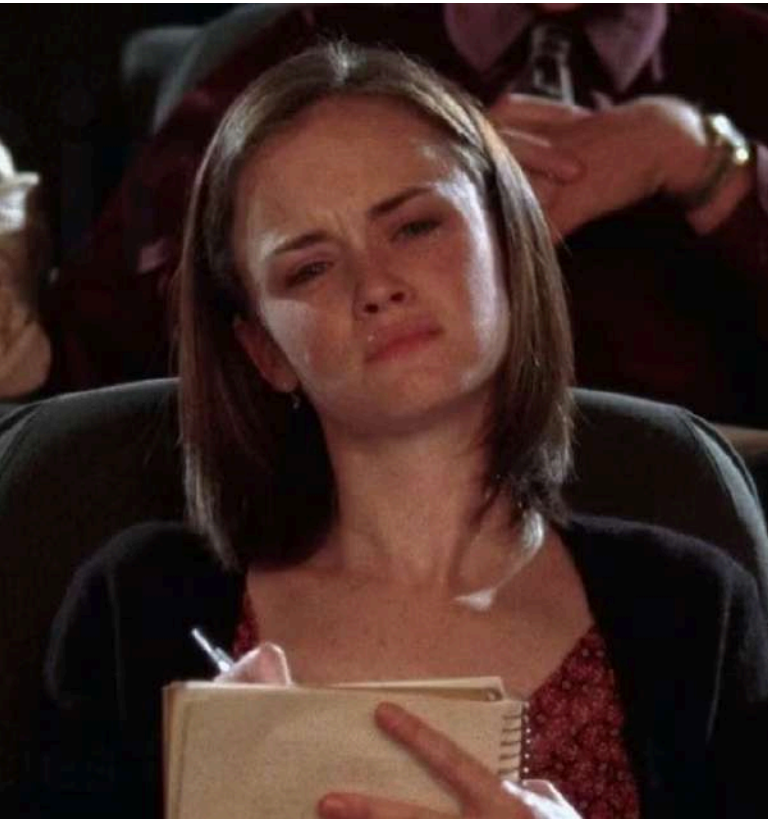

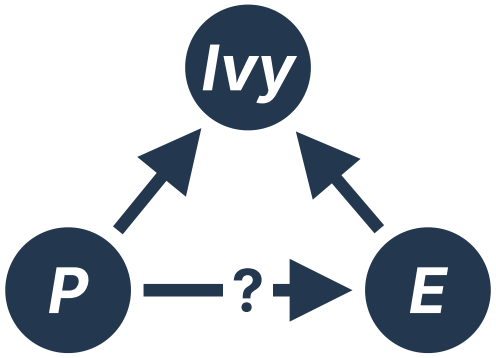

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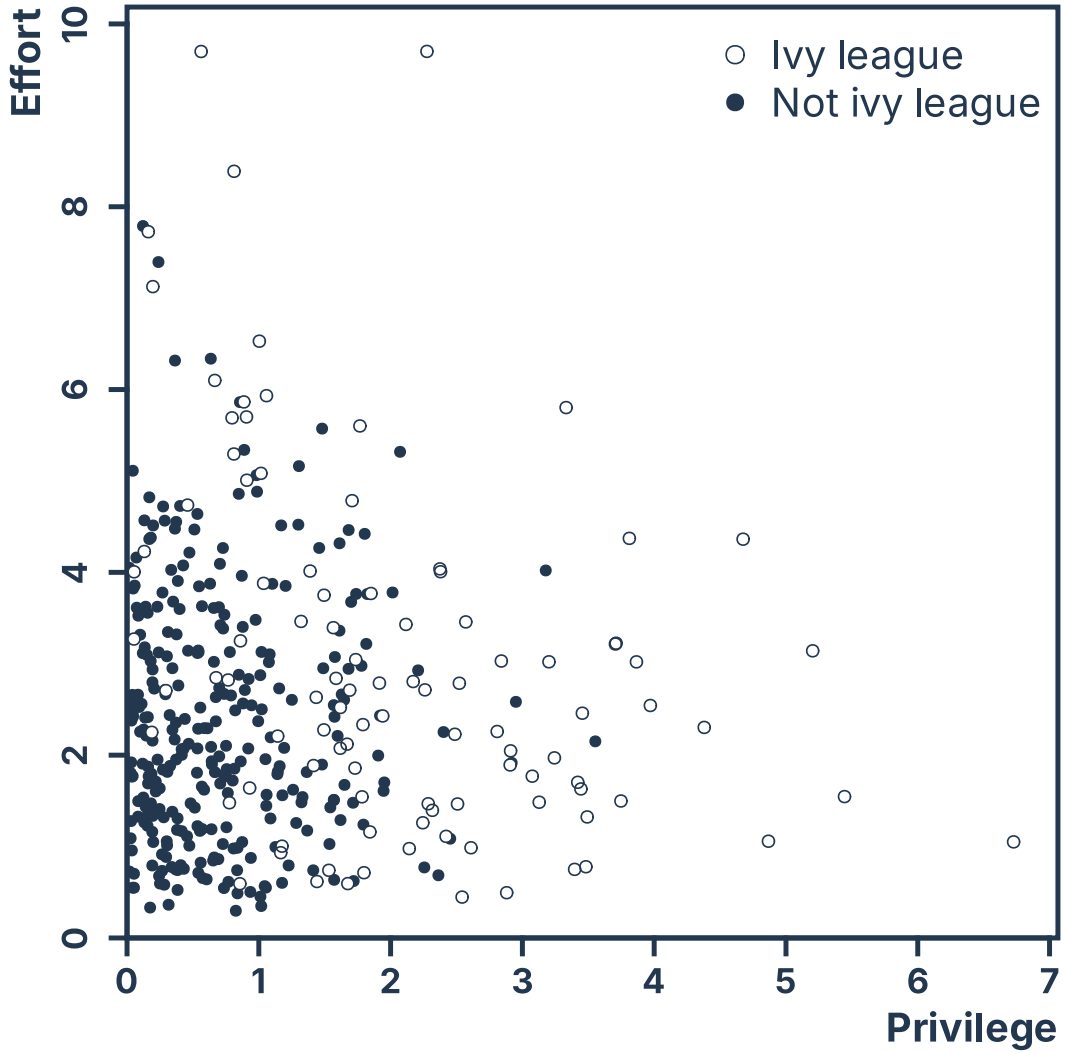

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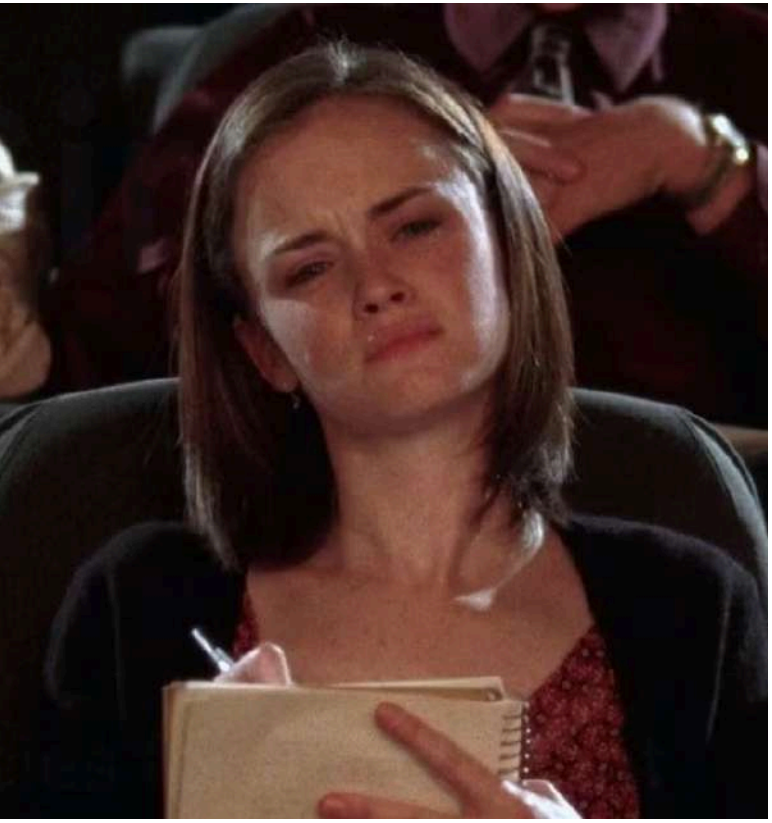

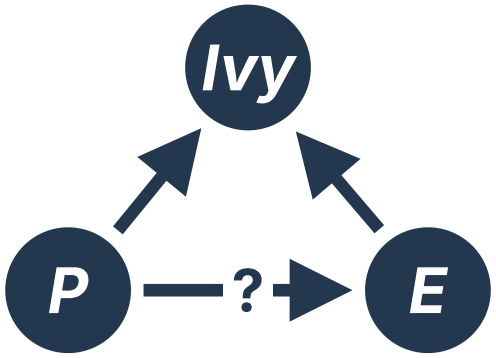

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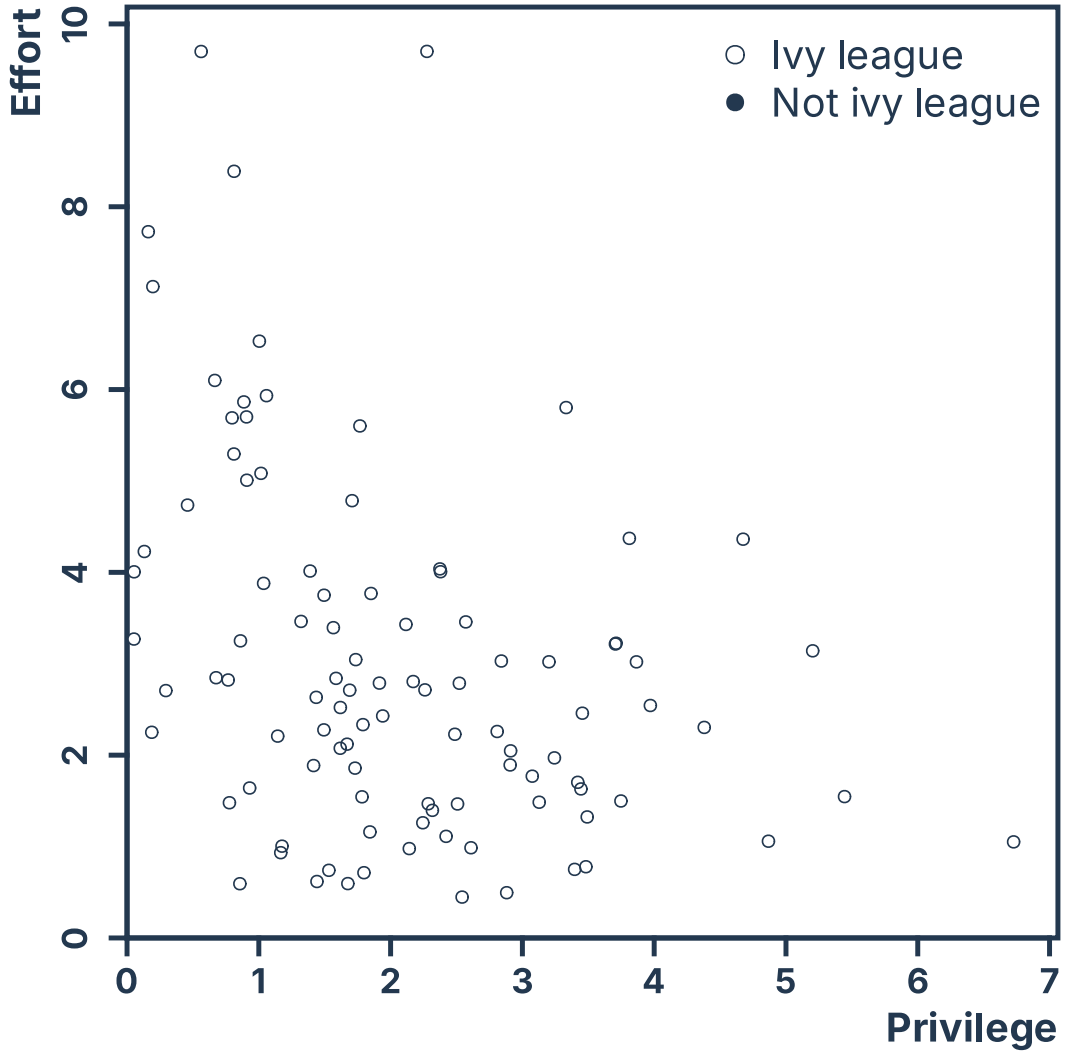

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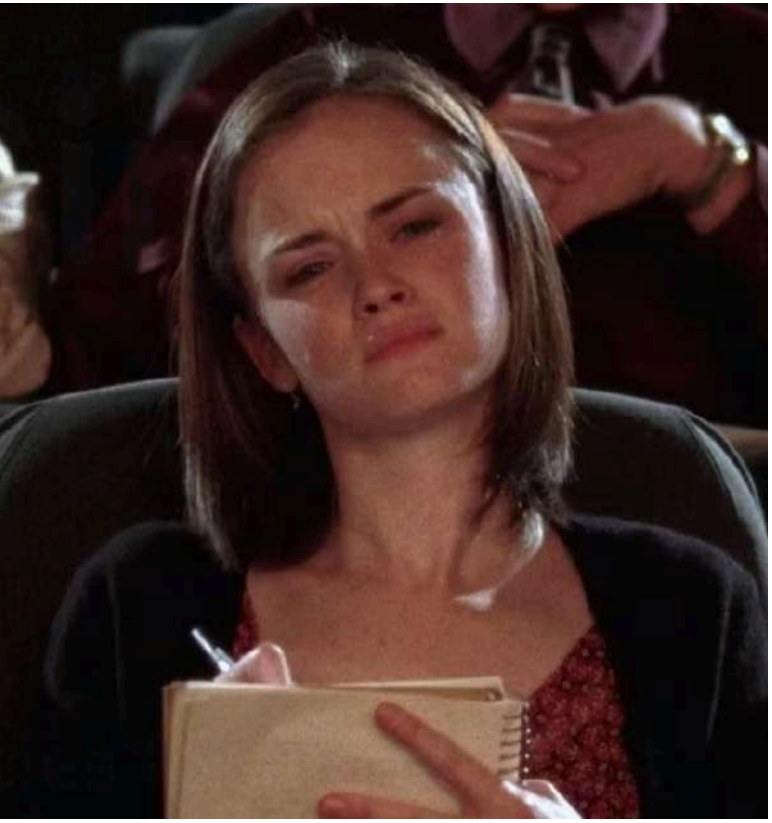

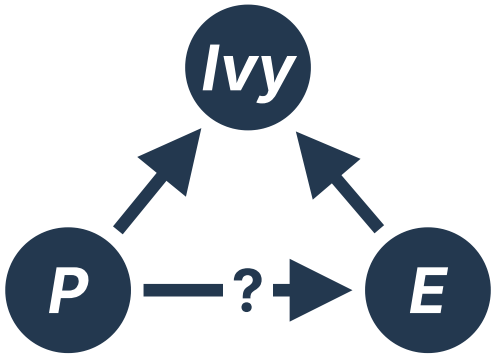

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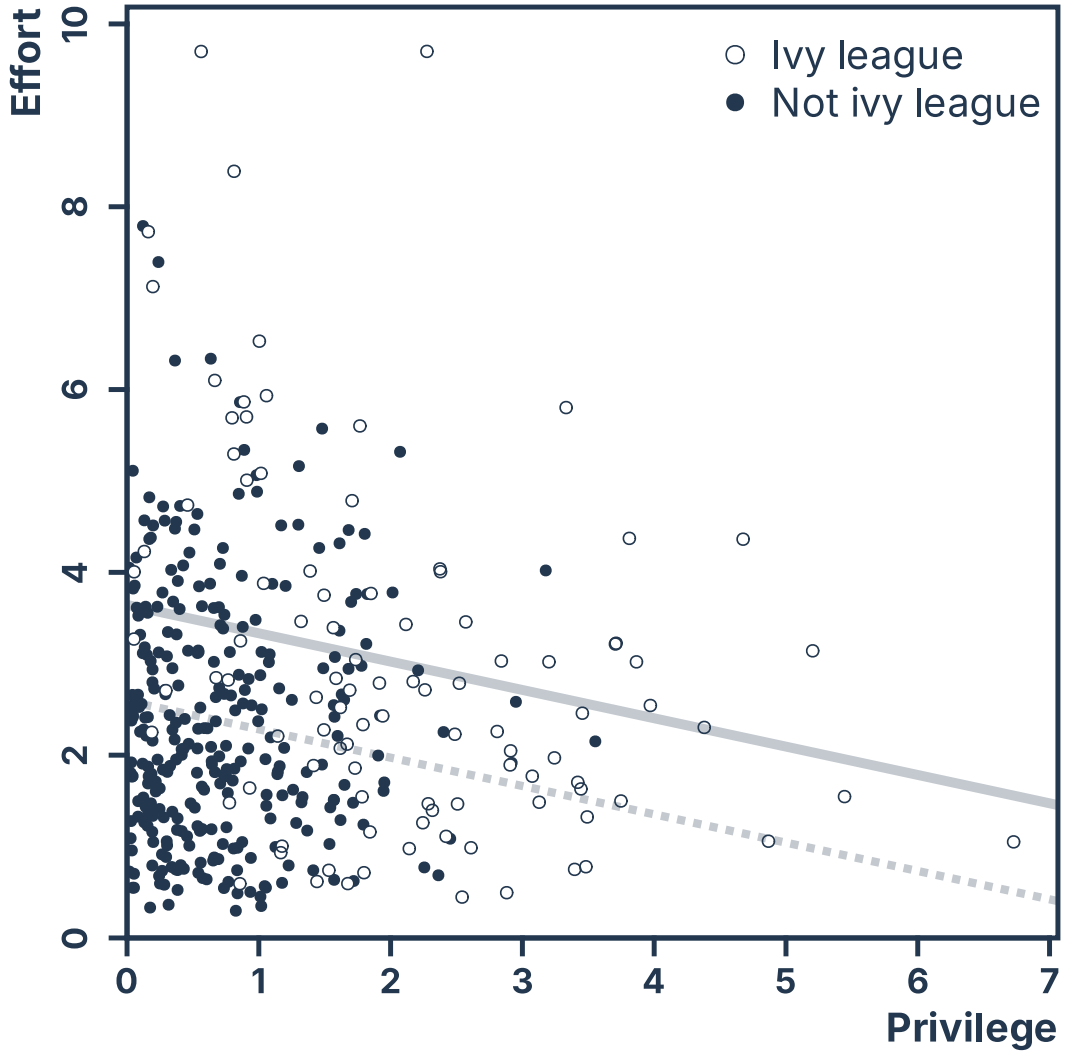

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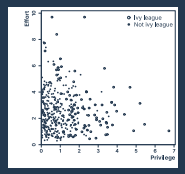


Figures by Peter McMahan (source code)

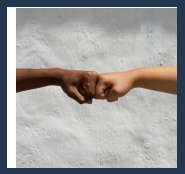
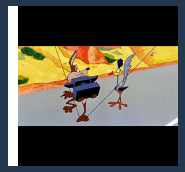
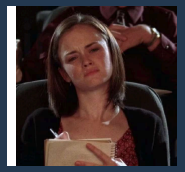


Photo by Arturo Añez on Unsplash



Road Runner and Wile E. Coyote © Warner Bros. Entertainment



Still from Gilmore Girls (2000)